NAVAL POSTGRADUATE SCHOOL
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THESIS

A SYSTEMS ANALYSIS OF THE
COUNTERFORCE POTENTIALS MODEL

by

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October 1982

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A Systems Analysis of the Counterforce Potentials Model

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ABSTRACT

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I. INTRODUCTION

A. THE NEED

The Land Forces Division (LFD) of the Office of the Assistant Secretary of Defense for Program Analysis and Evaluation (OASDPA&E) desires a quick time computer model to examine the effects of different U.S. weapon mixes against a Soviet force. They want the model to include both direct and indirect fire weapon systems for at least a division size combined arms operation. When the Land Forces Division examined the current models market they discovered that no model met their criteria. In addition they felt that the currently used firepower scores (FPS) based upon the product of lethal area times expected ammunition expenditures and the Anti-potential Potential (AP-P) eigenvalue approach did not contain enough battlefield interrelationships and weapon characteristics in their data to be useful in examining force mix relationships. Therefore, they were looking for an eigenvalue method which would employ more battlefield qualities.

The LFD contracted with the System Planning Corporation (SPC) for a model to accomplish their proposed force mix analysis. The SPC's proposal was the Counterforce Potential model written in August 1981 by F.W. Young and T.F. Hafer.

The question remains whether this model meets the needs of the LFD. This answer is the goal of this thesis.

B. BACKGROUND/HISTORICAL PERSPECTIVE

1. General Introduction

In a review of large scale modeling, the main methods were linear models of imputing values to weapon system types. Within these models, the subjective firepower score approach, the lethal area times the expected ammunition expenditure fire power score approach (which will be referred to as the product method), and the eigenvalue approach will be examined. In Chapter IV these methods will be used to compare the consistency of the Counterforce Potential (CP) results.

According to Lester and Robinson [1], a firepower score is a single number representing the military worth or capability of a particular weapon system. Commonly, it is used to develop a measure of combat potential of a force, which can be of any size usually greater than a division. This is achieved through the use of the firepower index. The index and score are not synonymous, although they are closely related. A score is a number assigned to a weapon system and an index is the linear sum of the scores within a unit. The score is also an input of a linear model, while the index is the scalar output of the same model.

The index is a linear combination of the firepower scores and represents the aggregation of all the weapon systems within the force. The general linear formula, as shown by James G. Taylor [2] is $I = \sum_i S_i X_i$. Here S_i represents the firepower score of a weapon of type i and X_i is the number of

weapon i that there are in the force. According to J.R. Bode [3], an index like the firepower score may be developed from a totally subjective base, the product method, a combination of the linear model with judgemental relationships (WEI/WUV), or the eigenvalue method.

2. Subjective Firepower Scores

a. General

In the subjective firepower score approach there are two methods which will be described, "subjective" and WEI/WUV.

The "subjective" method is a quite simple approach. A military committee subjectively assigns firepower score values to weapon systems in accordance with a given bounded scale.

A more complicated subjective approach is the WEI/WUV method. In this approach a committee of military personnel apply a subjective weighting (called a DELPHI number) to weapon characteristics in order to develop a score for a weapon in a particular family of weapons.

b. "Subjective" Firepower Scores

The easiest way of describing this methodology is through an example. A committee of military officers placed a subjective score on an entire weapon system. This committee was provided the bounded scale (0-100) and the systems to be scored. Their results are shown in Table 1.

TABLE 1

Subjective Firepower Score/Index

WEAPON	SCORE (S_i)	QUANTITY	S_iX_i
M16	1	100	100
M60 MG	3	10	30
Tank	19	10	190
Tow	10	5	50
Dragon	11	4	44
Howitzer	17	10	170
		INDEX:	584

c. WEI/WUV

Lester and Robinson [1] state that all of the weapons considered are divided into seven classes according to general functions and characteristics: portable small arms, vehicle mounted small arms, tanks, Armored Vehicles, anti-tank, artillery, and mortar. A standard weapon was picked in each family: M16, M113, M60A1, M551, TOW, 155mm Howitzer, and 107mm Mortar. Some versions allow the M16 to be the standard weapon for all the families so a better comparative relation can be made. For each family, a set of dominant characteristics was listed; for example: Tanks are defined by firepower, mobility, and survivability.

For each weapon a WEI is defined as the weighted sum of the dominant characteristics. J.R. Bode [3] gives an

equation for this as: $WEI = W_1C_1 + W_2C_2 + W_3C_3 + \ldots +$ where W_i are the subjective weights and C_i are the dominant characteristics. The W can be different for each i but $\sum_i W_i = 1.0$.

Each of the dominant characteristics is broken down to subcharacteristics and each of those is also subjectively weighted. The equation for each dominant characteristic is: $C_i = \sum_i d_i(Sc_i/Sc_{si})$ where the sum of the d_i equals one (1.0) and the Sc_i are subcharacteristics normalized to the standard weapon.

For example, the score for a tank (M60A1) would be developed as follows for the dominant characteristics of firepower (F), mobility (M), and survivability (Z):

Firepower $F = f(\text{lethality, ammunition, aux-weapons})$

$$F = .4(L/L_s) + .3(A/A_s) + .3(aW/aW_s)$$

Mobility $M = f(\text{speed})$ where: speed (Sp)

$$M = 1.0(Sp/Sp_s)$$

Survivability $Z = f(\text{speed, armor thickness})$

where: armor thickness (T)

$$Z = .5(SP/SP_s + T/T_s)$$

where all the d_i are subjective weights which can be different for each subcharacteristic.

Let the M16 be the standard weapon.

Let $L = 200$, $L_s = 1$, $A = 48$, $A_s = 18$, $aW = 3$, $aW_s = 1$, $Sp = 30$, $Sp_s = 1$, $T = 5$, $T_s = 1$; then

$$F = .4(200) + .3(48/18) + .3(3) = 81.7$$

$$M = 30$$

$$S = .5(35) = 17.5$$

Thus,

$$WEI(Tank) = .6(81.7) + .2(30) + .2(17.5) = 58.52 \text{ or } 59.$$

The WEI/WUV system is a combination of subjective weights applied to quantified characteristics. A completed WEI/WUV system based on the M16 being the standard weapon is displayed in Table 2.

TABLE 2

WEI/WUV Score/Index

ITEM	WEI	QUANTITY	S_iX_i
M16	1.0	100	100
M60 MG	8.0	10	80
Tank	59	10	590
TOW	40	5	200
Dragon	35	4	140
Howitzer	113	10	1130
		INDEX:	2240

3. Product Method

a. General

A more widely used approach is one in which the score equals the product of lethal area and expected ammunition expenditures. The single round lethality is found in tables and is calculated from empirical data. The ammunition expenditures (rate of fire) is a numerical constant and can be found in tables or in technical manuals of the weapons. Most of the empirical data for relating lethalities of point and area fires were gathered in WWII. Lester and Robinson point out the relationships and speculations that exist with the WWII data [1]. Questionable areas include age of the data, new versus old weapon types, and relevancy to current trends in tactical warfare. These questionable areas make the use of this empirical data for current model usage dubious.

b. Product Method Methodology

The data values (computed by the product method) were extracted out of FM 105-5 [4] at 300 meters for the same scenario as above. The scores were precomputed by the field manual. The index is computed using the form: $I = \sum_i S_i X_i$.

c. Rank Orderings of Weapons

The significance of the comparison of two methods is their order ranking of the weapon systems, based upon their S_i, score. These rankings are shown in Table 4.

TABLE 3

Product Firepower Scores

WEAPONS	SCORE (S_i)	QUANTITY	S_iX_i
M16	1	100	100
M60 MG	6	10	60
Tank	34	10	340
TOW	60	5	300
Dragon	50	4	200
Howitzer	100	10	1000
		INDEX:	1694

TABLE 4

Order Rankings

	SUBJECTIVE	WEI	PRODUCT
HIGH	Tank	Howitzer	Howitzer
	Howitzer	Tank	TOW
	Dragon	TOW	Dragon
	TOW	Dragon	Tank
	M60 MG	M60 MG	M60 MG
LOW	M16	M16	M16

The ranking differences prove that problems in imputing relative values to weapon systems can develop depending upon the chosen imputing method. A totally consistent method for imputing value and achieving rankings is presently non-existent.

These order rankings are important for sensitivity and force mix analysis. Therefore it is critical to have a method that is proven acceptable.

When these methods are used for both forces, friendly (blue) and enemy (red), we obtain two indices. The force ratio is the ratio of these opposing indices. Although in most models the force ratio is a value used to predict battle outcomes we shall consider the static force ratio as our measure of effectiveness (MOE) as it is in the Counter-force Potentials model. Thus the force ratios must be consistent among the methodologies. This will be examined in Chapter IV.

4. Eigenvalue Method

a. General

The third method to be examined is the eigenvalue method. This method was developed for modeling in the early 1970's, and had its origin in the IDAGAM model. It computes firepower score values and thus can be used to provide force ratios.

The eigenvalue method says that the value of a type i weapon system is equal to the value of everything it can kill. Thus the eigenvalue approach is more scenario dependent than the previously examined methods. The weapon quantities in the force level vectors and the attrition coefficients for the scenario affect the values of each weapon.

An advantage to this approach is that it eliminates the weighting factor from the subjective method. It additionally allows the force levels and the scenario to impact upon the resulting values of the weapons.

b. Methodology

According to Anderson [5], the principle of the Anti-potential Potential method is that the value of each combatant of type i is proportional to the total value of enemies it kills. The following methodology derivation for the eigenvalue method is given:

Let X be a vector which represents the number of systems of type i in the blue force: $X = (X_1, X_2, X_3, \ldots, X_m)$. Let Y be a vector which represents the number of systems of type j in the red force: $Y = (Y_1, Y_2, Y_3, \ldots, Y_N)$. Now, S_i^X and S_j^Y are unknowns corresponding to the values of type i and j systems for X and Y, respectively. Let a_{ij} be a m x n matrix which represent the rates type j systems kill type i systems and b_{ji} be a n x m matrix which represents the rate type i systems kill type j systems. Recall this method is scenario dependent, and this is represented by the input a_{ij} and b_{ji} matrices which vary with the scenario. The equation forms are:

$$C_X S_i^X = \sum_{j=1}^{n} b_{ji} S_j^Y$$

$$C_Y S_j^Y = \sum_{i=1}^{m} a_{ij} S_i^X$$

where:

C_x and C_y are proportionality constants;

a_{ij} and b_{ji} are attrition coefficient matrices;

S_i^X and S_j^Y are weapon values.

Combining yields:

$$C_x C_y S^X = (A\ B)^T S^X.$$

This form is an eigenvalue problem of the form:

$$\lambda Z = MZ$$

where

λ is the eigenvalue and M is a matrix.

Where:

$$C_x C_y = \lambda^*$$

$$S^X = Z$$

$$(A\ B)^T = M = ((a_{ij})(b_{ji}))^T$$

The eigenvalue method is well defined under the following conditions:

(1) If $(AB)^T \geq 0$ then there exists for one real λ, λ greater than 0, a corresponding eigenvector which is greater than or equal to 0.

(2) λ^* is the largest eigenvalue and corresponds to $C_x C_y = \lambda^*$.

(3) S^X is the eigenvector corresponding to λ^*.

Early criticisms of this method include:[1]

(1) the inability for the military leaders to influence the relative value and ranking of the weapon systems.

(2) interactions of the matrices a_{ij} and b_{ji}, which can lead to S^x and S^y that are paradoxical.

The System Planning Corporation picked the eigenvalue approach and sought to improve the calculations of the values of each weapon system by reflecting more combat processes on the a_{ij} and b_{ji} matrices. Their product was the Counterforce Potentials model.

C. COUNTERFORCE POTENTIALS MODEL FLOW

The Counterforce Potentials model is designed to provide a static measure of effectiveness for alternative blue (US) weapons forces against a user specified red (Soviet) force. The methodology is responsive to any scenario involving armor oriented combined arms threat.

---

[1]Hartman, J.K., Unpublished classnotes on Mathematical Models in Combat, Naval Postgraduate School, 1982.

1. The Approach

The model assumes that ground forces fight to take or hold terrain with direct fire weapons as their primary weapon system. Another assumption is that the typical battle is characterized by a continuous exchange of indirect fire in support of short term direct fire engagements. The approach has four submodules, shown here in Figure 1.

2. Counterbattery Exchange

The Counterbattery exchange is between indirect fire weapons systems only. Its purpose is to assess the casualties, damages, and suppressive effects of firing batteries against their possible target batteries. It examines the general support activities of each force and uses the Quickie Artillery model [6] to perform the calculations. The assessment tables are used with specified user allocations of counterbattery and countermaneuver battery ammunition to measure the degradation of allocated fires for the countermaneuver exchange based on the counterbattery exchange results.

3. Countermaneuver Exchange

The countermaneuver exchange considers the indirect fire systems against the direct fire systems. It examines the direct support activities of the forces and again uses the Quickie model [6]. Fractional casualties, damage, and suppression of direct fire weapons are calculated from density (rounds per unit area) and the mean area of effectiveness (MAE) of indirect munitions which are user specified.

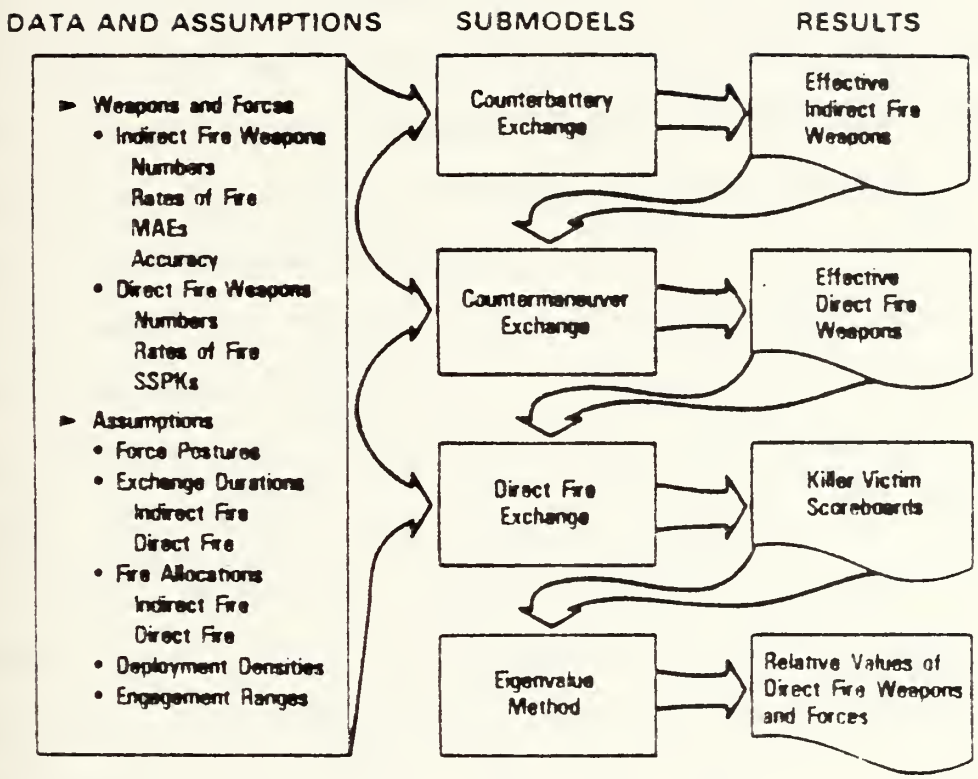

Figure 1. Methodology Flow

4. Direct Fire Exchange

The direct fire exchange is between opposing direct fire weapons that remain after the effects of the countermaneuver exchange are assessed. The 1 on 1 kill potential of each weapon system in this model is defined as a function of single shot kill probability (PPSK), probability of engagement and rate of fire. Average 1 on 1 kill potentials are calculated for each direct fire weapon pair based upon user specified distributions of ranges and force postures. Total kill potentials, P_{ij} which correspond to the a_{ij} and b_{ji} matrices used in the eigenvalue method, are calculated in proportion to the remaining number of direct fire weapons after the countermaneuver exchange and in accordance with optional user-specified allocations of fire, f_{ij}, among potential target types. This results in a series of killer-victim scoreboards that show the potential number of kills for each type of weapon against each weapon type in the force. These are potential kills because they do not reflect the size or the employment of the enemy in the kill rates. An example will be shown in the analysis of the direct fire module in Chapter II.

5. Eigenvalue Method

From the killer-victim scoreboards, P_{ij} and $\hat{P}_{ji}$, an eigenvalue method is used to compute the relative value of each direct fire weapon system and the total of each direct fire force. This eigenvalue approach assumes the value of

each weapon is proportional to the total value of the enemy weapons it is capable of killing. An arbitrary weapon is selected to be the base weapon with an assigned eigenvalue of 1.0. Using this method, the total value of the force is computed by summing the values of direct fire weapons that survive the opposing Countermaneuver exchange. A force ratio is then defined as the total blue force value divided by the total red force value. The value of the ratio is the measure of effectiveness, MOE, of the model. The objective of alternative force mixes is to achieve the highest ratio against a constant user specified enemy force.

D. INTRODUCTION SUMMARY

Major innovative changes in the calculations of the force ratio are incorporated in this model. Although the analytic forms of the eigenvalue method are quite similar to the original AP-P methods, the methods of obtaining the inputs and the type weapon systems used in the method are different. The eigenvalue method in the CP model includes only the direct fire weapons where past models have included both direct and indirect fire weapons within their AP-P approach. The counterforce potential's methods of incorporating the indirect fire weapon are quite different from previous models, although easily followed. It is critical with these new techniques to insure that the model has not missed the intent of the requested information in its attempt to provide numerical solutions to the user. These differences incorporated in the Counterforce

Potentials model will be examined to determine if they are valid in the attempt to answer the force mix questions.

E. PREVIEW OF ANALYSIS

The flow of the submodules through their analytic forms will be examined in Chapter II. The underlying assumptions and the analytical form used to achieve the outputs, which are used as inputs elsewhere within the model, will also be viewed. Emphasis will be on the individual analytic form as an input-output device for the model. The theoretical model forms will be analyzed for their use in realistic combat fire-power mix analysis.

In Chapter III the submodules will be linked and examined for their inter-relationships. These submodules are the Counterbattery, Countemaneuver, Direct fire, and Eigenvalue modules. The order of implementation of the submodules will be examined to determine if they provide realistic outputs supporting tactical employment within combat modeling. The goal will be to examine if the model, as linked together, can be used as intended by the Land Forces Division, i.e., to obtain the optimal force mix for the U.S. forces for given scenarios.

In Chapter IV will be an examination and comparison of the Counterforce Potential model to the previously described methods of subjective FPS, product FPS, and an earlier eigen-value approach (similar to IDAGAM). A specific scenario will be given and the force ratios for each method will be calculated,

examined, and compared. Sensitivity analysis of the question whether to increase tanks or artillery will also be examined and compared. The goal is to check for consistency between these methods.

Chapter V will be the conclusions and VI the recommendations to the Land Forces Division based upon the analysis.

II. DESCRIPTION OF ANALYTIC FORMS

In this chapter the analytic forms used within each submodule will be described, examined, and analyzed. This chapter will flow in the same sequence shown in Figure 2. The equations will be treated independently within their respective submodule.

A. COUNTERBATTERY EXCHANGE

1. Rounds Fired in the Indirect Fire Submodules

The total number of rounds fired, N_i, is computed for each particular type of indirect fire weapon.

a. Assumptions

All available weapon fire at a sustained rate of fire over a given fire period.

b. Model Form: $N_i = W_i \times R_i \times T_i$

Where:

N_i = number of type i rounds fired;

W_i = number of available type i weapons;

R_i = maximum sustained rate of fire of weapon i;

T_i = duration of the indirect fire period.

c. Analysis

All the inputs to N_i are user inputs. The user inputs are W_i, the weapons, R_i, the rate of fire, and T_i, the period. The inputs of W_i and T_i are scenario dependent as each

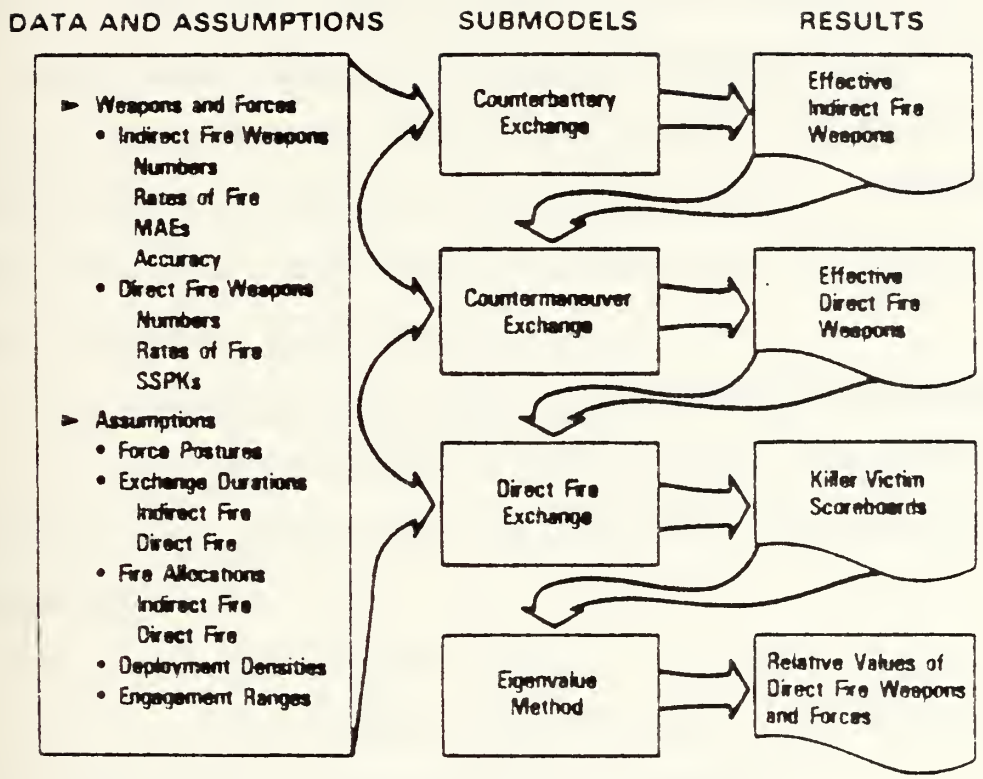

Figure 2. Methodology Flow

reflects changes to the force and the fire period respectively. The maximum sustained rate of fire for weapon i, R_i, is a value which is obtained in a testing environment. R_i can be found in the technical characteristics data of each weapon in its respective technical manual. Testing for each weapon differs somewhat in conditions thus each test reflects different effects on the maximum rate of fire. There exist many other areas which affect achieving the maximum rate of fire--in particular, target acquisition. Rather than to try to subjectively measure or quantify these effects on R_i, it seems better to use the tested rate that is in the manuals. The results of using these constants are an optimistic number for the rounds fired by weapon i.

2. Allocation of Rounds to the Indirect Fire Submodules

The total number of rounds fired are allocated to each of two indirect fire submodules: counterbattery and countermaneuver.

a. Assumptions

(1) User can satisfactorily allocate the fraction of rounds for the counterbattery exchange.

(2) The total number of rounds is equal to the sum of the rounds expended in the two submodules.

b. Model Form: $N_i^{CB} = (N_i)(f_i^{CB})$

$$N_i^{CM} = (N_i)(1 - f_i^{CB})$$

where:

N_i^{CB} = number of type i rounds allocated to counterbattery;

f_i^{CB} = user-specified counterbattery allocation for weapon type i ($0 \leq f \leq 1$);

N_i^{CM} = number of type i rounds allocated for countermaneuver fires.

c. Analysis

The user input of the f_i^{CB} is critical and needs to have a more sound basis from empirical data than to be subjectively obtained. The computational form is a straight forward fractional form. It is important for the user to have a basic idea of the allocation for the fractional amount of rounds fired for counterbattery. Most artillery officers claim that there is no straightforward way to allocate fractional fires since the counterbattery mission is highly dependent upon the enemy weapons being detected and engaged [9]. It seems possible to measure this fractional allocation, f_i^{CB}, in one of these three presented manners.

(1) Assume that the indirect weapons have only two missions, counterbattery and countermaneuver and that these are sole dedicated missions. These missions are assigned at battalion level either being Direct Support (for countermaneuver) or General Support (for counterbattery). Then f_i^{CB} can be estimated as the sum of the General Support weapon i's divided by the total number of weapon i in the force.

(2) The user's specification of susceptible weapons to counterbattery fire can be used to set up an allocation scheme. The fraction f can be the sum of the susceptible weapon i divided by the sum of these weapons plus the direct fire target force.

Formula:

$$f_i^{CB} = (\sum_i Sc_i I_i)/(\sum_i Sc_i I_i) + (\sum_i Q_i)$$

where:

Sc_i = the percent of type i weapons that are susceptible to counterbattery fire;

I_i = the number of indirect fire weapons i;

Q_i = the number of direct fire weapons i.

(3) Subjective assignment by the user as currently done in the model.

In the above allocation schemes flaws still exist. In case (1) each battery of Direct and General Support will not always fire sole dedicated missions but will fire both counterbattery and countermaneuver missions. In case (2) the examination requires intelligence information which probably will not be available during advance planning. In case (3) the subjective input may give radically different results. Subjectivity should be excluded from this model for it to be an improved method.

3. Average Volleys Fired by Each Battery Weapon

a. Assumptions

A proportionality relationship exists in the determination of volley quantities.

b. Model Form: $V_{ij} = (N_i^{CB} \times M_j)/(M_i \times Q_T)$

Where:

V_{ij} = average number of type i volleys fired against each opposing type j battery;

N_i^{CB} = number of type i rounds allocated for counterbattery fires;

M_i = number of weapons per type i firing battery;

M_j = number of weapons per type j target battery;

Q_T = the total number of indirect fire weapons in the opposing force that are susceptible to counterbattery fire.

c. Analysis

This quantity is an expected value for volleys at a J target battery. There are no priorities of fires and each battery is designated by user inputs as susceptible or not susceptible. The number of volleys is influenced by the user specification of susceptibility. As an expected value there is no provision for a massing or concentration of fires at a target.

The model has tables for the 1,2,4,6,8,10,12,16, 20, and 24 volley effects for damages, casualties, and suppression. Thus if the computational volley, V_{ij}, is not one

of the tabulated volleys than an interpolation for effects must be used in the next sequence.

4. Fractional Effects in Counterbattery

The fractional effects for damages, casualties, and suppression are calculated for the value of V_{ij}, the average volleys fired by each battery weapon.

a. Assumptions

(1) The effects are deterministic events for a given number of volleys.

(2) Effects of damages, casualties, and suppression can be expressed by the same deterministic functional form with different coefficients.

(3) The effects are approximated by an exponential expression.

(4) If V_{ij} is not one of the tabulated V then it will be between two which are thus interpolated for the subsequent effects.

(5) The maximum number of volleys is 24.

b. Model Form: f_{ij} (C,D, or S) $= 1-\exp(-(av+b))$

Where:

f_{ij} = fractional loss (D,C, or S);

v = the number of volleys of interest;

$a = \ln(1-f_H) - \ln(1-f_L)/(1-V_H)$

$b = \ln(1-f_L) + aV_L$

and v_H and v_L are the nearest tabulated number of volleys ($v_L \leq v \leq v_H$) with corresponding fractional effectiveness values of f_L and f_H.

c. Analysis

The exponential expression tends to be the intuitive approach since as the number of volleys is increased the fractional effects, f_{ij}, of damages, casualties, and suppression also increase. In examining the form, it is important to verify that the interpolation is correct for the cases where $V = V_L$ and $V = V_H$.

(1) Case 1: $V = V_L$

$$f_{ij} = 1-\exp(-(av+b)) \text{ ? } f$$

$$f_L = 1-\exp(-(aV_L + \ln(1-f_L) + aV_L))$$

At this point, the aV_L's would be expected to cancel and the final exponent of e should be $\ln(1-f_L)$ and not $-\ln(1-f_L)$. In this case the sign of b in the original form should be negative. The form should be: $f_{ij} = 1-\exp(-(av-b))$.

(2) Case 2: $V = V_H$

$$f = 1 - \exp(-(av + b)$$

$$f = 1 - \exp(-(av_H + \ln(1-f) + av_H)$$

Again at this point the av_H's are expected to cancel and the exponent of e should be ln(1-f). In order to achieve this the original form should be: $f_{ij} = 1-\exp(-(av-b))$.

As shown in the two cases a sign error exists in the documentation. The results using this form will yield incorrect interpolations and erroneous effects. If this error does not exist in the computer code or when it is corrected, then results yielded would be a correct interpolative effect of the number of volleys fired.

5. Total Fractional Losses in Counterbattery

a. Assumptions

(1) The fractional casualties, damages, and suppression are independent for each weapon i and for all susceptible weapon j's.

(2) $0 \leq C_{ij}, D_{ij}, S_{ij} \leq 1$.

(3) Weighting factor in the interval [0,1] must be applied for suppression to account for the length of firing period in regards to the length of the suppression (default of .5 is assumed).

(4) For reasoned suppression recovery is in seconds.

(5) For unit disruption recovery in 15-20 minutes.

(6) Model assumes unit disruption only.

b. Model form: $L_j = 1 - \prod_{i=1}^{I} (1-C_{ij})(1-D_{ij})(1-w_s S_{ij})$

Where:

L = the total combined fractional losses to weapon j from all the effects for i type of firing weapons;

C_{ij} = fractional casualties from weapon type i;

D_{ij} = fractional damages from weapon type i;

S_{ij} = fractional suppression from weapon type i;

w_s = suppression weighting factor.

c. Analysis

The assumption of independence is highly suspect. For an individual weapon type i firing at target j the fractional effects are obtained. It seems reasonable that an artillery round that impacts which causes damages should also cause casualties to personnel for the weapon damaged. It also seems reasonable that if a round impacts causing casualties that there is a good probability that it also caused damages to their weapon system. In the same sense the round which causes damages and casualties could suppress weapon and personnel near the impact. Thus it appears intuitive for a particular weapon i firing at a target j the effects of damages, casualties, and suppression are not independent but dependent events. It is not expected that the three effects would occur as independent points due to an impact point. A more likely expression would be for the events to occur within a given area dependent upon each other.

The above argument holds true for the effects of successive volleys from a weapon i. A weapon firing volleys at the same target would cause the same relationship to the effects as the single round case. Thus the effects of weapon

i would still be considered dependent events. For example, if a forward observer reports the target is down the weapon discontinues its mission. This shows that a weapon fires are dependent upon the results of the previous rounds effects.

For the effects of different weapons firing at target j it appears reasonable to assume independence. The effect of the other weapon round will not affect the damages or casualty effects of the other weapon. The effects due to the round for suppression could be affected by the other weapon rounds and thus independence is not a good assumption for the suppression effects.

According to Fort Sill there exist later versions of the Quickie model [7] which overcome many of the statistical independence problems. The SPC employs an earlier version of the Quickie model [6] in which the independence assumptions are not overcome. The later version should be considered for inclusion in the model.

Suppression and the use of the suppression weighting factor in this model do not express the possible events relative to suppressive fires. In the Mitre Corporation's "Counterforce Campaign Analysis" [8], they express three events which could occur relative to suppressive fires.

(1) The unit receiving the fires ceases his mission and does nothing.

(2) The unit receiving the fires ignores the effects, buttons up, and continues his missions.

(3) The unit moves from the position receiving the fires and continues operations and missions.

These are reasonable events to consider for suppression. Considering these events as variables, the weighting factor encompasses too many variables to be estimated by a constant value for a given scenario. Since the events do not occur across the battlefield uniformly, it seems inappropriate to assign one number to represent all the events.

Combat Developments at Fort Sill treats suppression as a highly suspect area in modeling. Models based upon historical suppressive data may not be appropriate for the new and future weapon systems. It may be better to eliminate the suppressive effects until a better scheme is developed to express the events and the effects adequately.

6. Total Effectiveness of the Indirect Weapon Type

a. Assumptions

(1) Rate of losses over the firing period is constant.

b. Model Form: $E_i = 1 - L_i/2$

Where:

E = average fractional effectiveness of weapon i;

L = total losses of i due to all j indirect fire weapons.

c. Analysis

Our value here is the average fractional effectiveness of the weapon type. There does not seem to be any suspect area in this modeling approach.

At the beginning of the time period there are 100% of the opposing indirect fire systems. Loss takes place at a constant rate over the period so losses equal the loss rate times the period. The average losses over the period are L/2. The effectiveness of the indirect fire systems is then, on the average, 1 - L/2. The counterbattery and countermaneuver exchanges occur over the same period of time so the average losses due to counterbattery are used as an input to the countermaneuver exchange.

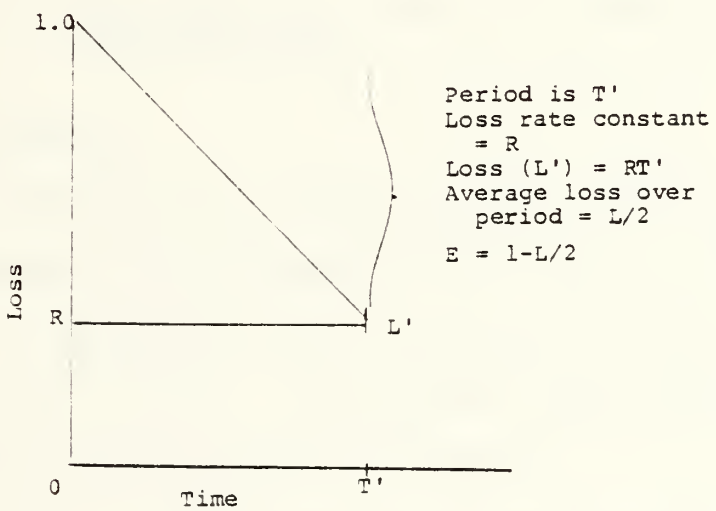

Figure 3. Fractional Effectiveness Due to Constant Loss Rate

B. COUNTERMANEUVER EXCHANGE

1. Calculation of the Number of Countermaneuver Rounds

a. Assumptions

Effectiveness of weapon i impacts upon its ability to fire rounds.

b. Model Form: $U_i = E_i N_i^{CM}$

Where:

U_i = number of countermaneuver rounds fired by type i weapons;

E_i = the average fractional effectiveness of the firing weapon type i;

N_i^{CM} = the number of rounds allocated for countermaneuver fires

c. Analysis

Previously the number of rounds for the countermaneuver exchange was expressed as $N_i^{CM} = N_i(1 - f_i^{CB})$. This was the optimistic number of rounds allocated to countermaneuver. N_i is equal to $N_i^{CM} + N_i^{CB}$. The adjusted form reflects the degradation due to the attrition effects.

It is not clear why the total degradation factor is applied solely to countermaneuver except for modeling convenience. The counterbattery exchange fires the optimistic number of rounds and countermaneuver exchange fires adjusted number of rounds. The order of combat firing sequence is significant and will be addressed later but let it suffice to to say that countermaneuver should receive some degradation but not the entire factor.

2. Calculation of Fractional Losses in Countermaneuver

a. Assumptions

(1) Loss is a deterministic function of density and MAE.

(2) The function is approximated by an exponential expression.

(3) The density is equal to total rounds/total area.

(4) Total area is equal to the number of direct fire weapons in the target force divided by the assumed average density of the same weapons in the formations we expect will be used.

b. Model Form: $f_{\substack{ij \\ C,D,\text{ or } S}} = 1 - \exp(-(d_i(MAE)))$

Where:

f_{ij} = fractional losses in damages, casualties, or suppression;

d_i = density of i's rounds per meter squared (m^2)

MAE = mean area of effectiveness of round i;

where:

$$d_i = U_i/A$$

U_i = countermaneuver rounds for weapon i;

A = target area

c. Analysis

The basic theoretical form reveals as density, MAE, or both increase with the other factors held constant as the effectiveness increases. This is what we would expect to theoretically occur. The exponential expression is intuitively pleasing.

According to T. Hafer,[2] for a given area of size A the loss rate is independent of the size of the target force. For example, in an area of size A the rate of losses to tanks is 25% regardless of the number of tanks in the area. Thus in the basic form the approach employs area type fires for the indirect fire systems against the direct fire force.

The System Planning Corporation provides the user an expression for the area A when the user is in an attacking posture. The expression is: $A = T/Z$. In this expression, T is the number of direct fire weapons and Z is their assumed density in their target formations. This approach assumes a great deal of intelligence information for the firing systems, which would not reasonably be the case for an entire force.

The emphasis on density and MAE still imply area type fires over a given area. There is no provision for massing of fires, the most significant generator of combat fire power provided by the artillery against a target [9]. The form must be sensitive to this attribute of artillery.

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[2]Hafer, T., Interview, Naval Postgraduate School, July 1982.

3. Total Fractional Loss of Each Direct Fire Weapon Type

a. Assumptions

(1) Fractional losses are independent and deterministic functions.

(2) Independence by weapon and target types.

b. Model Form: $L_j = 1 - \prod_{i=1}^{I} (1-C_{ij})(1-D_{ij})(1-w_s S_{ij})$

Where:

L_j = total combined fractional losses to the j direct fire weapon from all the effects for type I firing weapons;

C_{ij} = fractional casualties from weapon type i;

D_{ij} = fractional damages from weapon type i;

S_{ij} = fractional suppression from weapon type i;

w_s = suppression weighting factor.

c. Analysis

The analysis is generally as expressed in Section A.4.c.

As before, the independence question is suspect. The direct fire weapon systems have a different relationship between the man-machine systems than do the indirect fire systems. The direct fire system is self-contained. Thus damages, casualties, and suppression are dependent upon the round from system i and not independent for all firing weapons at the direct fire system.

The arguments for the single round and the single as described previously are even more valid against the

direct fire system than they were against other indirect fire systems. The effects are suspect as independent events.

4. Total Fractional Effectiveness in Countermaneuver

a. Assumptions

(1) Effectiveness is a deterministic function and has values between (0,1).

(2) Losses are a deterministic function with value between (0,1).

b. Model Form: $E_i = 1 - L_i$

Where:

E_i = fractional effectiveness of each type direct fire weapon;

L_i = total combined fractional loss to the i direct fire weapon from all the effects for j type firing weapons.

c. Analysis

It is important to relate why effectiveness is now equal to 1-L whereas before it was equal to 1-L/2. Before the losses at the end of the period were not the objective. The average losses were used to get the average effectiveness of indirect fire systems for the countermaneuver exchange. Now the indirect fire systems are engaging the direct fire system. The model assumes that all the indirect fire exchanges occur prior to the direct fire engagement. Thus the objective is the total losses at the end of the indirect fire period. At the end of the period there are L losses. Thus the resulting effectiveness, E, is 1-L.

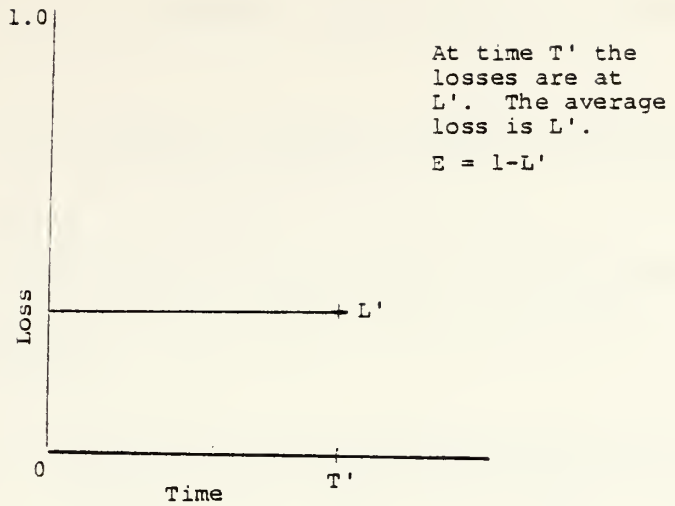


Figure 4. Fractional Effectivness Due to Losses

C. DIRECT FIRE EXCHANGE

1. Direct Fire Kill Potentials, K_{ij} (1 on 1)

a. Assumptions

(1) Kill potentials are a measure of the number of targets (of a particular type) that a surviving direct fire weapon would be able to kill during a single direct fire engagement. It is an i on j engagement pair.

(2) Engagement period is assumed to be one (1) minute.

(3) Shooters posture, attacking or defending is not considered.

(4) Cardiod hit distribution is used for single shot kill probability.

(5) Rayleigh distribution is used to express the probability of an engagement at range R.

(6) All weapon target pairs (i versus j) are considered.

(7) Assume a single direct fire engagement period.

b. Model Form: $K_{ij} = T_D \int_0^{\infty} N(R) P_k(R) \rho(R) dR$

Where:

K_{ij} = the 1 on 1 kill potential of i direct fire weapon type against j target weapon type;

T_D = length of the direct fire period in minutes;

$N(R)$ = the rate of fire at range R (shots per minute);

$P_k(R)$ = single-shot kill probability at range R;

$\rho(R)$ = probability that an engagement will occur at range R and

$$\int_0^{\infty} \rho(R) dR = 1.$$

$N(R) = 60/(t_d + t_f)$, where:

t_d = detection time in seconds;

t_f = firing time in seconds.

c. Analysis

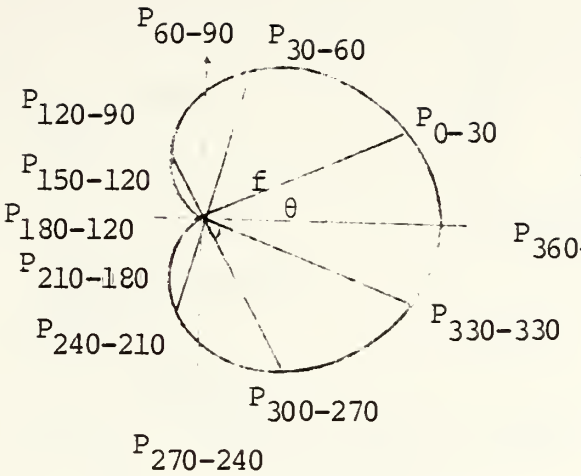
The direct fire kill potential is a value assigned to each weapon target pair. This value is a measure of the number of particular type targets that a surviving direct fire weapon would be able to kill during a single period of direct fire engagements. In the theoretical form the kill

potentials are a function of time, range, Pssk, probability of engagement, and rates of fire.

The firing time t_f is a user input. The firing time consists of the total time between shots to guide the round to the target (as in the TOW), to reload, and to aim at another target in an assumed target rich environment. The definition of t_f must vary from weapon to weapon to reflect whether any of the functions can be accomplished simultaneously. For example a tank can reload and aim at the same time but a LAW cannot. It appears with this definition that the user may have a hard time making a good estimate for t_f. It would be an advantage for the user if this data were obtained and tabularized for use within the model.

The detection time parameter, t_d, is meant to be a variable which reflects time to detect in a target lean environment so systems can employ and take advantage of their target acquisition capabilities. Currently the model is not using t_d as a function of acquisition and range. The parameter t_d is significant in the calculation of N(R). The parameter must reflect target acquisition as a function of the type system and the range. Like t_f, t_d should be provided as real data available for use in the model.

The probability of single shot kill at range R is tabularized. It is derived from a cardiod hit distribution. This particular distribution is used often in the modeling of single shot kill probabilities and is accepted in the modeling community.



$$P_i = P_{\theta_2-\theta_1}$$

$$P_{Kill|Hit} = \sum_{i=1}^{12} P_i \cdot P(Kill|aspect\ angle\ i) \qquad P_{SSK} = P_{Hit}\, P_{Kill|Hit}$$

Figure 5. Cardioid Hit Distribution

It is a realistic expression of single shot kill probabilities.

The $\rho(R)$, probability of engagement at range R, has been characterized as a Rayleigh distribution. Young and Hafer relate they chose the Rayleigh due to its following advantages: (1) it represents the distribution of ranges between a point in the cartesian coordinate system and a set of normally distributed points and (2) it is characterized by a single parameter which facilitates rapid sensitivity analysis.

The distribution appears to reflect more than just a probability of engagement. It reflects for a given α parameter, α relating to the type terrain over the range, the most

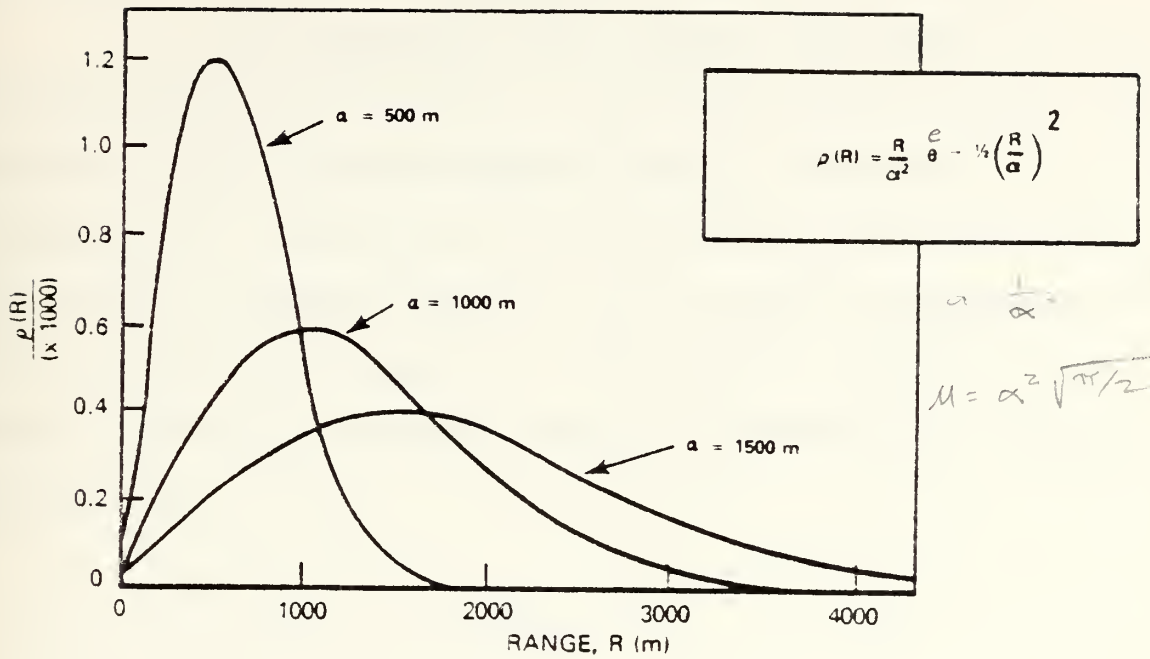


Figure 6. The Rayleigh Distribution

likely range of engagement. The peaks of the distribution reflect the highest probability of an engagement. For example in the case of α = 500 meters (which relates to mountainous, or heavily forrested terrain, a firer would more likely engage a target at a range of 500 meters than at a range of 1000 meters or 100 meters.

$$\rho(500) = .0012$$

$$\rho(1000) = .00054$$

$$\rho(100) = .00039$$

This analysis shows that in this type of terrain the firer's most likely engagement range is 500 meters.

The individual weapon target kill potential, K_{ij}, is calculated for each type i versus each type j system. K_{ij} reflects a one on one by weapon type kill potential. K_{ij} assumes that i directs all of its fires at each target j for the entire engagement period. K_{ij} is then the potential kill rate of weapon i at target j. For example, assume we have tanks (i) firing at saggers, BMPs, and troops the j targets. Then for K_{ij}:

TABLE 5

1 on 1 Kill Potentials

K_{ij}	Sagger	Troops	BMP
Tanks	.6	.8	.3

These represent possible 1 on 1 kill potentials of the tank against the j targets. This example illustrates the highest 1 on 1 kill potential is troops. As this example is continued in the analysis the significance of the values of K_{ij} will be seen to be modified by other factors.

2. Killer-Victim Scoreboards, Total Kill Potentials P_{ij}

a. Assumptions

(1) Scoreboard values represent the number of target kills by type target for each firing weapon type.

(2) The allocation fraction f_{ij} is dependent upon the option the user selects.

(3) There are three option allocations for f_{ij} numbers, threat, ability to kill the threat.

(4) No other option schemes for f_{ij} have been developed.

(5) Number of effective i weapons is Q_i times E_i.

(6) Allocation options are provided to the user for each weapon type.

b. Model Form: $P_{ij} = f_{ij} \times Q_i \times E_i \times K_{ij}$

Where:

P_{ij} = total kill potential of weapon i against weapon j;

f_{ij} = fractional allocation of the force's available weapon-on-minutes of type i against type j targets;

Q_i = quantity of weapon i in the force;

E_i = fractional effectiveness of weapon i from countermaneuver exchange;

K_{ij} = 1 on 1 kill potential of weapon i against target j assuming all fires that are at j.

The following are allocation options for f_{ij}:

$$f_{ij} = Q_j / \sum_{i=1}^{J} Q_j$$

$$f_{ij} = Q_j \hat{K}_{ji} / \sum_{j=1}^{J} Q_j \hat{K}_{ji}$$

$$f_{ij} = Q_j \hat{K}_{ji} K_{ij} / \sum_{j=1}^{J} Q_j \hat{K}_{ji} K_{ij}$$

where:

f_{ij} = fractional allocation of firing type i against target type weapon j;

Q_j = the number of type j target weapons in the opposing force, unless the 1 on 1 kill potential is zero, then Q_j equals zero;

K_{ij} = the 1 on 1 kill potential of firing weapon against the target weapon;

$\hat{K}_{ji}$ = the 1 on 1 kill potential of the target weapon against the firing weapon.

c. Analysis

The f_{ij} factor represents the fraction of force available weapon-minutes of type i potential allocated against type j targets. The factor can be expressed in terms of the number of available target weapons, their proposed threat, or i's ability to kill j's threat. The forms of f_{ij} are linear fractions, where the sum over j of f_{ij} is one (1.0) for all i. These schemes are reasonable allocation methods. The forms are simple yet adequate to achieve the fractional results desired. Although priorities of direct fire are not addressed it does appear as though the three allocation methods cover the possible priorities. The only questionable area in the

allocation methods is why Q_j was used and not $Q_j \times E_j$ product. It appears that the fractional effectiveness of the J force should play into the allocation scheme of the i potential. The E_j factor could alter the allocation schemes and perhaps have six available methods.

The factor Q_i, the available i systems, and E_i, the fractional effectiveness of the i force, are used together to give the realization that not all weapons are at 100 percent ability. Their product will yield the usable weapon count of i type weapons. It is this factor that is considered with the allocation factor f times the one on one kill potential to achieve the potential number of target kills by type, P_{ij}, for each firing weapon type. Thus the product is the total potential of type i weapon against type j targets for direct fire weapon systems.

The P_{ij} is the total force potential of type i weapons against type j targets. Continuing the example from the one on one kill potentials, the calculation for P_{ij} for 100 Tanks (system i) with an effectiveness, E, of .75 using the form $P_{ij} = K_{ij}Q_iE_if_{ij}$ would be as shown below in Table 6. Let there be an opposing force consisting of 50 Saggers, 200 Individual Troops, and 40 BMPs. Using the option of allocation by threat where $\hat{K}_{ji}$ is (.8,.01,13) for the kill rates of the sagger, troops, and BMP against tanks. The threat option uses

$$f_{ij} = \frac{Q_j \hat{K}_{ji}}{\sum_j Q_j \hat{K}_{ji}} .$$

TABLE 6

Total Kill Potentials

	Sagger	Troops	BMP
Tank K_{ij}	.6	.8	.3
f_{ij}	.625	.03	.19
$K_{ij}f_{ij}$	.375	.024	.057
P_{ij}	28.1	1.8	4.3

It is significant to point out that although the K_{ij} against the troops is the highest potential 1-1 kill rate, that the troops receive the lowest total kill potentials. The fractional allocation, f_{ij}, is the contributor to this change. The low threat by the enemy troops led to the tanks potential kills going to the Saggers and BMPs.

The P_{ij} are called potentials because the number of opposing weapons by type may be less than the total number of potential kills calculated.

D. EIGENVALUE METHOD

1. Relative Values of the Forces

a. Assumptions

(1) The value of each weapon is proportional to the value of the weapons it is capable of killing.

(2) C_x equals β_x, and C_y equals β_y.

(3) Beta, β, is the constant of proportionality introduced to create simultaneous equations from the proportional relationships.

(4) One weapon type is assigned an arbitrary value of 1.0.

(5) In normal eigenvalue methods there are no guarantees of unique solutions.

(6) By eliminating indirect fire weapons we can guarantee unique solutions.

b. Model Form: $V_i^B \propto (1/Q_i^B) \sum_{j=1}^{J} P_{ij} V_j^R, V_j^R \propto (1/Q_j^R) \sum_{i=1}^{I} \hat{P}_{ji} V_i^B$

Where:

V_i^B = the value of a single type i Blue weapon;

V_i^R = the value of a single type j Red weapon;

Q_i^B = the initial number of type i weapons in the force;

Q_j^R = the initial number of type j weapons in the force;

P_{ij} = the total kill potential of i against j;

$\hat{P}_{ji}$ = the total kill potential of j against i.

With proportionality constant beta, β, yields:

$$\beta V_i^B = (1/Q_i^B) \sum_{j=1}^{J} P_{ij} V_j^R$$

$$\beta V_j^R = (1/Q_j^R) \sum_{i=1}^{I} \hat{P}_{ji} V_i^B$$

c. Analysis

The theoretical form is consistent with the form discussed by B. Anderson in the IDAGAM model [5]. The method performs as described in the background of Chapter I. The lack of indirect fire weapons in the eigenvalue computations is of concern but will be addressed in Chapter III.

2. Indices and Force Ratio

a. Assumptions

A linear model relationship exists.

b. Model Form: FR = (Blue Force value)/(Red force value)

Where:

$$\text{Blue force value} = \sum_{i=1}^{I} Q_i^B V_i^B$$

$$\text{Red force value} = \sum_{j=1}^{J} Q_j^R V_j^R$$

c. Analysis

The forms are consistent in their linear relationships with the forms expressed by Prof. J.G. Taylor and the IDAGAM model [2,5]. Although consistent with previous relationships it is questionable whether these forms exhibit the necessary characteristics to be useful in force mix analysis.

According to Lester and Robinson [1], a critical shortcoming in the firepower score method with constant weapon values results from the linear relationships $I = \sum_i S_i X_i$. In

the model the change in I is proportional to changes in numbers but real world evidence indicates that changes in the mix of weapons has a strong influence on the relative value of each weapon type. In the model the value of the 1000th weapon is the same as the 1st, 10th, and 100th weapon of the same type. This exhibits no diminishing marginal returns as weapons are added [1,3]. A saturation point would be expected whereby the addition of more weapons would no longer contribute to combat potential and then the model would exhibit diminishing marginal returns. The diminishing marginal returns effect should exist in the model.

The value V in the Counterforce Potentials model is an implicit function of allocations, effectiveness, weapon quantities, and one on one kill potentials. An increase in the number of direct fire weapon i affects not only the total kill potentials i on j, P_{ij}, but also j on i, $\hat{P}_{ji}$, and thus the values of V_i^B, V_j^R for all i,j in the Blue and Red forces are also affected. This is significantly different from previous firepower scores and this characteristic is valuable. Thus all the relative values may change for both the red and blue force with just the change in one weapon i's quantity. The saturation and the diminishing marginal returns issue will be investigated in Chapter IV.

The force ratio is used to compare the total blue force value to the total red force value. Inprevious dynamic combat models the force ratio is used to predict damages,

casualties, and maneuver distances. The force ratio here is a static comparison of force values. The ratio form indicates that the weapon values must be ratio scale numbers. It does not seem unreasonable to assume an absolute zero point exists for all weapon values.

A difference relationship of blue and red values seems reasonable to investigate as an alternative to the ratio method. An example will best illustrate the investigation.

Case 1: Let $V^B = 10$, and $V^R = 30$. The force ratio is .3 and the difference measured by $V^B - V^R$ is -20. In case 2 let the blue value be 12 and the red value 33. The force ratio is .3636 but the difference is -21. In the difference relationship the greatest value is better thus case 1 would be chosen. In the force ratio method case 2 would be chosen. The force ratio tends to make more sense to the military mind whose normal military training causes him to think in terms of ratios and not differences.

The force ratio tends to be the better approach as the measure of effectiveness used to quantify the impact of increases or decreases in weapon numbers for a given scenario.

III. METHODOLOGY FLOW

In Chapter III the input-output flow between the sub-modules will be examined in relationship to the goal of the model. The flow will be consistent with Figure 7.

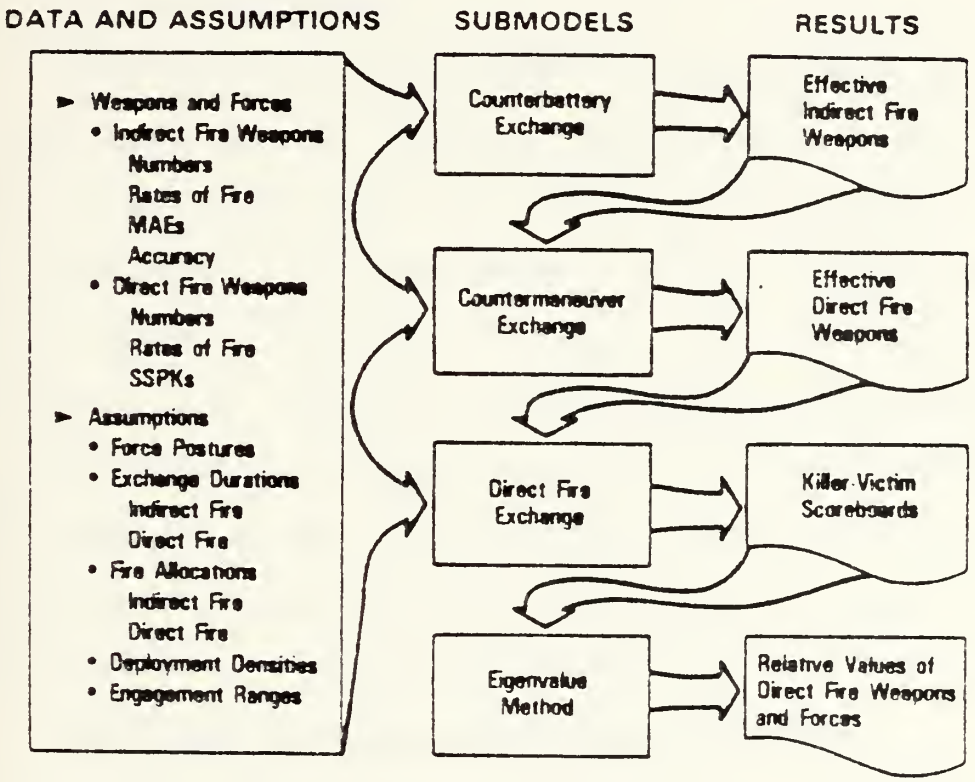


Figure 7. Methodology Flow

The goal of the model as related by Mr. Hap Miller[3] and Mr. Tom Hafer[4] is to enable the decision makers to analyze the

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[3]Miller, Hap, Interview, Naval Postgraduate School, April 1982.

[4]Hafer, T., Interview, Naval Postgraduate School, July 1982.

force mix changes against a constant opposing force in order to determine if the best force mix structures of US forces have been achieved. The measure of effectiveness, MOE, for this is the force ratio, the model's final output of the last submodule.

A. COUNTERBATTERY EXCHANGE

1. Input-Output Flow

The key input and output of this submodule are the N_i^{CB}, the input for the number of rounds fired in this submodule, and E_i, the output for the fractional effectiveness of the indirect fire weapon types. N_i^{CB} is solely a function of user inputs: the number of weapon i (W_i), the rate of fire of weapon i (R_i), the length of the indirect fire period (T_i), and the fractional allocation of fires to counterbattery (f_i^{CB}). The fractional effectiveness of weapon i, E_i, is calculated as one minus the product of fractional losses to weapon i and is carried to the next submodule for use as an input.

2. Problems in the Counterbattery Submodule

a. No Degradation of Rounds Fired in Counterbattery

During the 30 minute indirect fire period the optimistic number of rounds to be fired is all expended. Although attrition is taking place to the opposing susceptible indirect fire systems during the period the number of rounds fired by i at j, or j at i remains fixed throughout the exchange. All remaining submodules take into account degradation

or attrition in their exchange. The methodology should reflect the on-going attrition in a degradation of the number of rounds expended in the counterbattery exchange.

b. Counterbattery Submodule Proceeding Countermaneuver

This proposed organization is somewhat contradictory to the logical flow of combat. Counterbattery missions firing first are only valid when firing preparatory fires preceeding an attack [9] and depend heavily on intelligence information and use area fires. Since this model employs a weapon susceptibility to counterbattery fires it does not seem to be reflecting the preparatory fires but the fires directed at thwarting the enemy's firing of missions which would prohibit our maneuver forces from performing their mission [9]. Normally during a direct fire battle the opposing forces fire indirect fires at the maneuver forces. These firing systems become subject to detection and, if detected, will be engaged by the other indirect fire systems. Thus, in most cases, the counterbattery exchange would occur after the start but during the countermaneuver exchange.

c. User Specification of Susceptibility to Counterbattery Fire

The user's designation of the indirect fire weapons as susceptible or not susceptible to counterbattery fire does not account for all the possible cases mentioned for the reason to fire counterbattery. In most cases the indirect fire systems become detected and engaged by opposing indirect fire systems. The probability of detection (based on rounds fired

per weapon) and the conditional probability of engagement given a detection should be employed in lieu of user specification. Range could be a determining factor in that systems out of opposing firing range would not be considered in the counterbattery exchange. All other systems not affected by the range should be considered using the probability of detection and the conditional probability of engagement given a detection.

d. The Meaning of the Fractional Effectiveness, E_i

The fractional effectiveness of indirect fire weapons is dominated by the optimistic number of rounds expended and the user's specification of which weapons will receive counterbattery. This allows the user to directly influence the entire exchange.

B. COUNTERMANEUVER EXCHANGE

1. Input-Output Flow

The key input for this submodule is the fractional effectiveness of each indirect fire weapon, E_i, from the counterbattery exchange and the output is the fractional effectiveness, E_i, of each direct weapon. The input is used to degradate the number of rounds expended in the countermaneuver exchange. The output is carried to the direct fire exchange as an input.

2. Problems in the Countermaneuver Exchange

a. The Countermaneuver Exchange Comes After Counterbattery

As stated in the previous section the countermaneuver exchange should not logically follow the counterbattery exchange.

b. The Countermaneuver Receives the Entire Degradation of Rounds

In this section the entire degradation is applied prior to the exchange beginning. In most cases the countermaneuver exchange would begin its fires at full strength. As attrition occurs, due to counterbattery, the number of rounds is reduced. Perhaps a discrete time interval which reflects attrition to counterbattery and thus reduces rounds applied to both exchanges would be a better reflection of the combat abilities of indirect fire weapons.

c. No Concentration of Fires

The most unique and significant generator of immediate combat power is the ability of US fire support to mass their fires, i.e., many battalions firing accurately on the same target [9]. In this submodule there is no provision for this key mission of the artillery. The opposing weapons are not being exposed to mass fires therefore their fractional effectiveness is not reflecting total kill potential of the indirect fire systems.

d. No Attrition to the Indirect Fire Systems by Direct Fire Systems

The maneuver elements' mortars are considered an indirect fire system and are generally deployed in range of

direct fire systems. Their deployment makes them susceptible to direct fire weapons. This is not considered in the model. None of the indirect fire systems are ever attrited by direct fire weapons. Interactions of direct and indirect weapons is important for force mix questions, thus the absence of the impacts of direct fire on indirect fire weakens the force mix relationships.

e. The Meaning of the Output, E_i

The output E_i is the fractional effectiveness of direct fire weapons due to the opposing indirect fire weapons. This output is biased by having all round degradation applied to this exchange.

f. No Value Given to Indirect Fire Weapons

In order to adequately assess force mix questions the indirect fire systems should have a value assigned to them which can be related to the direct fire values.

C. DIRECT FIRE EXCHANGE

1. Input-Output Flow

The fractional effectiveness, E_i, from the Counter-maneuver exchange is used as an input in the calculation of the total kill potentials of the direct fire weapons. It is used with the one on one kill potentials, K_{ij}, calculated internally in this submodule, and the user specified allocation of fire option and the number of direct fire weapons i to get the output, P_{ij}, and the total kill potentials. These P_{ij} and $\hat{P}_{ji}$ are the matrices of the eigenvalue method.

2. Problems with the Direct Fire Exchange

a. No Foot Soldiers in the Model

There are currently no provisions for foot soldiers with their basic rifles and machine guns. A force mix structure in the US Army today is predominantly foot soldier oriented. Several divisions, the 7th and 9th, are unmechanized Infantry divisions and thus are even more dominated by the foot soldier. The model needs to be responsive for all divisions' force mix questions to be an effective analytic tool.

b. The Implied Use of T_D, the Direct Fire Period Length

It is implied in the methodology review that the value of T_D can be changed to reflect a longer fire period, thus affecting the kill potentials. In fact the value of T_D is doing no more than acting as a scale parameter applied equally to all P_{ij} and $\hat{P}_{ji}$. Eigenvalue methods are invariant to scale changes and thus a change in T_D has no effect on kill potentials or the subsequent weapon values.

D. EIGENVALUE METHOD

1. Input-Output Flow

The killer-victim scoreboards, the P_{ij} and $\hat{P}_{ji}$, are the key input to the eigenvalue method. These matrices reflect the scenario dependence of the eigenvalue methodology. They correspond to the a_{ij} and b_{ji} matrices explained earlier. The outputs of the method are the relative values of the weapons and the force ratio, the MOE.

2. Problems in the Eigenvalue Method

a. Indirect Fire Weapons in the Force Ratio Calculation

The force ratio does not explicitly reflect the value of the indirect fire weapons. Force mix questions concerning indirect fire systems are not explicitly addressed in this methodology.

b. Weapon Changes and Their Effects on Weapon Values

The effect that an increase in a weapon i has on weapon values and the resulting force ratio cannot be algebraically examined. The shortcomings previously mentioned of linearity and the failure to display diminishing marginal returns may occur in this model. This will be investigated numerically in Chapter IV.

E. THE FORCE MIX ANALYSIS

1. General

After the scenario is run the model proposes that by varying weapon quantities a new force ratio is obtained and compared to the original or previous force ratios. Within the model a change in weapon quantities affects not only the value of that weapon type but also the value of opposing weapon types. Although the opposing force size remains fixed in quantity its relative value changes. The force ratio is not just an improved blue force value being compared to the same red value but a completely new ratio of a new blue value to a new red value. A higher ratio is an improvement in the

force mix according to the model's methodology and assumptions. In most cases the user will have to reexercise the model in order to obtain the force ratio. The user requires a means in which to begin his force mix analysis. Haphazard changing of weapon quantities is not an acceptable method for force mix analysis.

2. Predetermined Weapons and Quantities in Force Mix Analysis

This is the simplest case, whe e the user knows prior to the original run which weapons and their quantities are to be examined. Inputing the changes in quantities and rerunning the model yields the new force ratio for the user to compare with the previous ratio. The user tests all his quantity changes and the best ratio implies the best force mix for that scenario.

3. Weapons Not Predetermined For Force Mix Analysis

a. Direct Fire Weapons Versus Direct Fire Weapons

In this case it seems feasible to allow the weapon value to determine the starting point for force mix analysis. Normally the weapon with the highest value would be a reasonable choice for further analysis. A high value implicitly implies survivability traits, thus it is a logical choice for force mix analysis.

b. Indirect Versus Direct Fire Systems

The indirect versus direct fire systems question, which is a significant decision for today's commanders, cannot be analyzed in a straightforward manner. The direct fire

weapons receive a relative value and thus can be rank ordered. The indirect weapons are not as well defined in the model and neither receive a value nor are rank ordered. A comparison of the perceived worth of an indirect weapon versus a direct fire weapon is not clearly available after the original run. There are no hints in the model as to the answer. It appears that the only available method is a system of trial and error where the user makes changes to weapons of the indirect and direct fire weapons and compares the resulting force ratios.

c. Indirect Versus Indirect Fire Weapons

The user is again faced with the same problem as before, where to begin? The model gives no relative value or rank ordering to the indirect fire weapons so the user does not know which indirect system is achieving the most success. Again the user seems to be forced to a system of trial and error as before.

4. Summary

If the model's methodology for calculating and using the force ratios is acceptable, then a methodology must be addressed for force mix analysis. In previous models employing imputing linear values to weapons, all weapons received relative values and were rank ordered. Thus the methodology for a situation where the weapons were not predetermined seems intuitive. The Counterforce Potentials model's treatment of the indirect fire systems in that they cannot be compared or ranked with the direct fire systems makes the force mix analysis difficult and unsystematic. The model's purpose is

to be useful in force mix analysis and currently it is not as useful as it could be expected to be. The model needs an internal methodology to approach the force mix analysis faced by the user. Once this methodology is provided for the user the model gives a methodology to compare the force mix relationships, the force ratio.

IV. COMPARISONS AND NUMERICAL SENSITIVITY ANALYSIS

A. INTRODUCTION

In this chapter, four methodologies of imputing value to weapon types are examined and compared. The four methodologies are the "subjective" firepower score method, the product firepower score method, the eigenvalue method, and the Counterforce Potentials method.

A hypothetical scenario is given and the force ratio is calculated for each methodology. A comparison of weapon order rankings and the force ratios is made to test the consistencies among the methodologies.

Numerical sensitivity analysis is applied to the Counterforce Potentials model to investigate the relationships of increases in direct and indirect fire systems to the changes in force ratio. Numerical sensitivity is also used to examine the force mix trade off analysis for the scenario. The force mix investigation includes both the direct and indirect fire weapon types.

B. THE SCENARIO

The scenario chosen for the analysis is a meeting engagement between an armored heavy U.S. division and two soviet motorized rifle divisions. The range for our engagement is between 1000-2000 meters. Allocation of fire is limited only to the fractional allocation by numbers of available targets

option. The weapon systems considered are limited to four types for each force.

TABLE 7

The Scenario

U.S. ITEM	Division Quantity	Soviet ITEM	Division Quantity
Tank M60A3	320	Tank T72	540
TOW(ITV)	50	BMP	108
Helo AH-1	18	Helo Hind	24
How. 155mm	36	How. 122sp	86

C. "SUBJECTIVE" FIREPOWER SCORES

A committee of military officers were given the scenario and asked to assign values to the weapon systems based upon a 0 to 100 scale. Their results were:

TABLE 8

Subjective Scenario Results

U.S.	Value	Soviet	Value
TOW	40.0	BMP	35.0
Howitzer	60.0	Howitzer	50.0
Tank	80.0	Tank	79.0
AH-1	90.0	Hind	88.0

1. "Subjective" Blue Force Value

The blue force value is equal to the sum of the weapon values times the quantity of weapon types ($V^B = \sum_{i=1}^{I} V_i^B \times Q_i^B$). V^B is 31380.

2. "Subjective" Red Force Value

The red force value (V^R) is formed the same way as V^B. $V^R = \sum_{j=1}^{J} V_j^R \times Q_j^R$. V^R is 52779.

3. Force Ratio

The force ratio is equal to the blue force value divided by the red force value (V^B/V^R). The force ratio equals .5945.

D. PRODUCT METHOD

The product method uses the product of lethality times the expected expenditures of ammunition to obtain the weapon values. The weapon values for the scenario were extracted from FM-105-5 which uses the product method to obtain the values of the weapon types [4].

TABLE 9

Product Scenario Results

U.S.	Value	Soviet	Value
Tow	60.0	BMP	50.0
Tank	32.0	Tank	34.0
Howitzer	50.0	Howitzer	20.0
AH-1	82.0	Hind	70.0

1. Blue Force Value

Using the formula $V^B = \sum_{i=1}^{I} V_i^B \times Q_i^B$, the value of the blue force is 16516.

2. Red Force Value

Using the formula $V^R = \sum_{j=1}^{J} V_j^R \times Q_j^R$, the value of the red force is 27160.

3. Force Ratio

The force ratio (V^B/V^R) equals .608.

E. EIGENVALUE METHOD

This method uses an APL program entitled "Poten" which is available at the Naval Postgraduate School.[5]

In order to use the program the attrition matrices a_{ij} and b_{ji} had to be developed. They were approximated by the product of acquisition rate, target selection fraction, and the single shot kill probability. The matrices used are:

TABLE 10

a_{ij} Matrix

	BMP	Tank	How	Hind
TOW	.025	2.8	0.0	0.0
Tank	.5	2.1	0.0	0.001
How	.05	1.8	1.8	0.0
AH-1	.05	2.52	2.6	1.0

[5]Hartman, James K., Unpublished class notes on Mathematical Models in Combat, Naval Postgraduate School, 1982.

TABLE 11

b_{ji} Matrix

	TOW	Tank	How	AH-1
BMP	.03	2.5	0.0	0.0
Tank	.5	2.0	0.0	0.0001
How	.04	1.5	1.8	0.0
Hind	.05	2.5	2.5	0.99

The weapon values obtained are:

TABLE 12

Eigenvalue Scenario Results

U.S.	Value	Soviet	Value
TOW	1.0	BMP	1.251
Tank	2.69	Tank	2.56
How	5.28	How	4.72
AH-1	14.67	Hind	14.87

1. Blue Force Value

Using the formula $V^B = \sum_{i=1}^{I} V_i^B \times Q_i^B$, the value of the blue force is 1366.08.

2. Red Force Value

Using the formula $V^R = \sum_{j=1}^{J} V_j^R \times Q_j^R$, the red force value equals 2281.91.

3. Force Ratio

The force ratio (V^B/V^R) equals .599.

F. COUNTERFORCE POTENTIALS MODEL

Allowing the model to run using the given scenario the following weapon values are obtained:

TABLE 13

Counterforce Scenario Results

U.S.	Value	Soviet	Value
TOW	.51	BMP	.26
AH-1	.66	Hind	.77
Tank	1.0	Tank	1.01

No values are assigned to the howtizers as they are an indirect fire system. Their effects are included in the values of the other systems.

1. Blue Force Value

Using the formula $V^B = \sum_{i=1}^{I} V_i^B \times Q_i^B$, the blue force value equals 356.6.

2. Red Force Value

Using the formula $V^R = \sum_{j=1}^{J} V_j^R \times Q_j^R$, the red force value equals 594.6.

3. Force Ratio

The force ratio (V^B/V^R) equals .60.

G. COMPARISON OF THE MODELS

The weapon order rankings and the force ratios are investigated for consistency.

1. Weapon Rank Orderings

The ordering are displayed in Table 14.

TABLE 14

Rank Order of Weapons

	Subjective	Product	Eigenvalue	Counterforce
HIGHEST	AH-1	AH-1	AH-1	Tank
	Tank	TOW	Howitzer	AH-1
	Howitzer	Howitzer	Tank	TOW
LOWEST	TOW	Tank	TOW	

The rank orderings are not consistent. The lack of consistency in weapon order ranking should alarm the reader.

Each method orders the weapons in accordance with different criteria. Examining the tank across the methods shows the tank varies from the lowest in the product method to the highest in the counterforce method. One should expect the methods to yield somewhat consistent results for the same scenario with the same weapons. Each method measures the value of a weapon in accordance with certain characteristics of the weapon. These characteristics are similar but are quantified differently. One should expect similar characteristics to be ordered consistently for the same weapon.

2. Force Ratios

The force ratios calculated from each methodology are displayed in Table 15.

TABLE 15

Force Ratio Comparisons

MODEL	FORCE RATIO
"Subjective"	.595
Product	.608
Eigenvalue	.599
Counterforce	.600

The force ratios appear to be consistent. The consistency reveals that although we may value systems differently according to different characteristics, the ratio of the two opposing forces is approximately equal among the methodologies for the sample scenario investigated.

The force ratio was examined to see if the Counterforce Potentials model yielded radically different results from other models already used in combat modeling. It is concluding that for the sample the model yields a result which is consistent with known models.

H. NUMERICAL SENSITIVITY ANALYSIS

1. General

In order to realize the difficulty in the Counterforce Potentials' theoretical sensitivity analysis, a

comparison is shown between it and the subjective WEI/WUV model. Figure 8 illustrates this comparison.

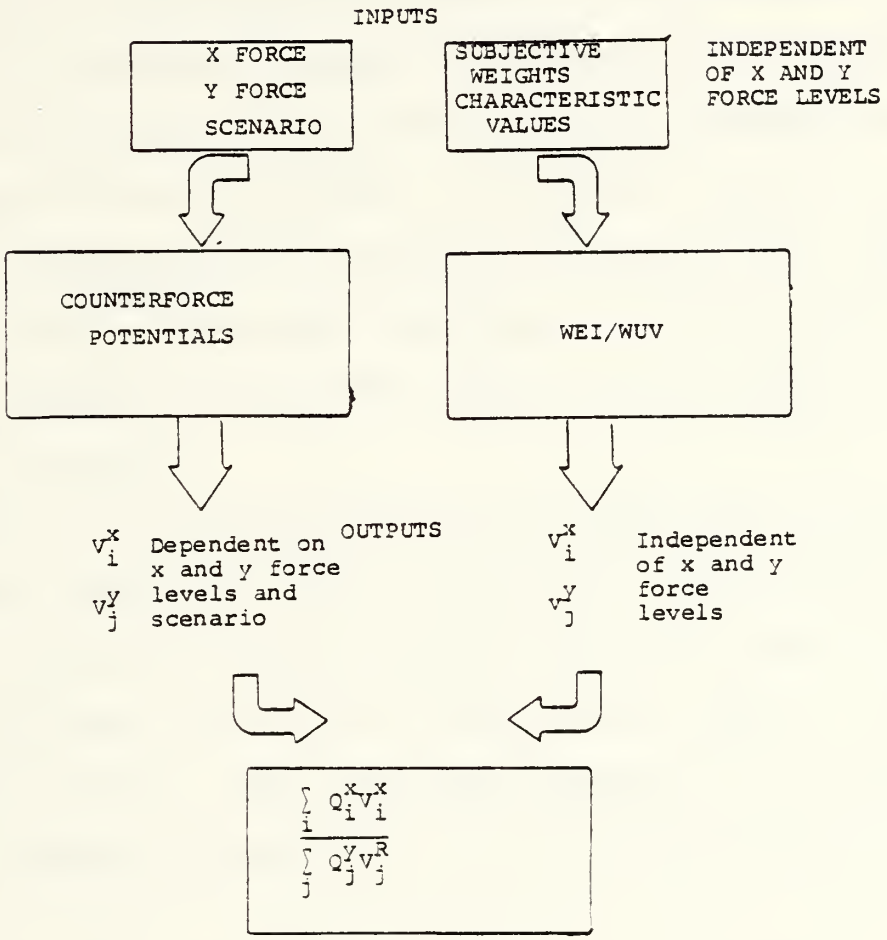

Figure 8. Models Comparison

As displayed in the figure the force ratio is formed from the same expression. The characteristics of the expression are critical to the force mix and sensitivity analysis.

In the WEI/WUV method the value of each weapon is independent of the quantity of weapons to be examined. The incremental increases in weapon numbers display the same increasing linear rate of the force ratio for increases of the same weapon by the same amount. The weapon with the highest individual value improves the force ratio the most. The marginal worth of the weapon increases is readily calculated but the linearity of the method makes its use suspect for force mix analysis.

In the Counterforce Potentials model the values of the weapons are dependent on the force levels and the scenario. Increases in a blue weapon quantity not only affect the value of the blue force but also can affect every weapon value in the opposing force. This makes any attempt at a straightforward algebraic sensitivity and force mix analysis extremely difficult.

For this reason numerical analysis is used to investigate the relationships to see if the model is linear and if it displays diminishing marginal returns.

2. Numerical Analysis

a. Force Ratio Versus Weapon Quantity Changes

From the scenario's basic weapon input quantities, an investigation into the relationships of all blue weapon

increases to the force ratio is made. In each case the weapon under investigation is increased by varying amounts. For each weapon increase, all other weapon quantities for the red and blue force remain constant. These weapon increases are displayed in Table 16.

TABLE 16

Force Ratio Versus Weapon Changes

Type Weapon	Quantity	ΔQ	Force Ratio	Δ Force Ratio	Slope (ΔQ/ΔFR)
Tanks	320	--	.60	--	--
	326	6	.61	.01	.00166
	338	12	.64	.03	.0025
	374	36	.72	.08	.0022
	640	320	1.38	.66	.002
	1000	360	2.17	.79	.002
	10000	9000	18.26	16.09	.002
TOW(ITH)	50	--	.60		
	53	3	.61	.01	$.00\overline{33}$
	56	3	.61	.00	.00
	68	12	.63	.02	.0016
	100	32	.70	.07	.002
AH-1	18	--	.60	--	--
	24	6	.61	.01	.00166
	36	12	.64	.03	.0025
Howitzer	36	--	.60	--	
	42	6	.60	0	0
	54	12	.60	0	0
	72	18	.61	.01	.0003
	90	18	.61	0	0

ΔQ = difference $Q_{NEW} - Q_{NEW-1}$; ΔFR = difference $FR_{NEW} - FR_{NEW-1}$

In Table 16, the slope (change in force ratio/change in weapon quantity) is examined because the slope represents the rate at which the output (force ratio) changes due to the input (weapon) changes. The rate of change for each increase of the same weapon is approximately the same. As displayed in the graphs (Figures 9-12) the relationships appear to be linear.

It is critical to point out that the Counterforce Potentials model rounds the force ratio to the nearest 100th. In the case of the howitzers this rounding makes the graph almost useless for examination.

These graphs tend to indicate constant returns to scale and show the model fails to exhibit diminishing marginal returns. Diminishing marginal returns is a property where as the quantity of a weapon is increased, the rate of increase of the FR is not constant, but rather diminishes. Ultimately a point might be reached where the force ratio actually decreases for further weapon increases. This has been called the saturation point of a weapon. This property is a must for force mix analysis. This model fails to display diminishing marginal returns.

Linear models do not possess this property. In linear models weapon increases always result in force ratio increases. Thus the effect of one weapon dominating the force mix is highly probable. Linearity is a shortcoming noted in force mix analysis. This model is linear.

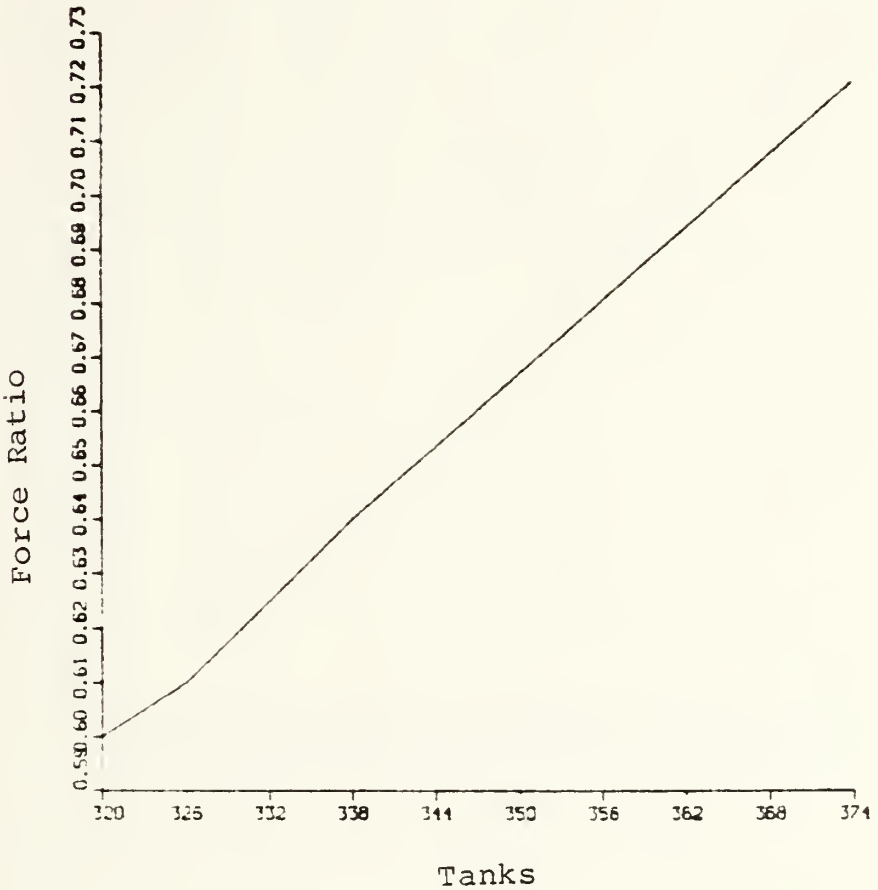

Figure 9. Force Ratio Versus Tanks

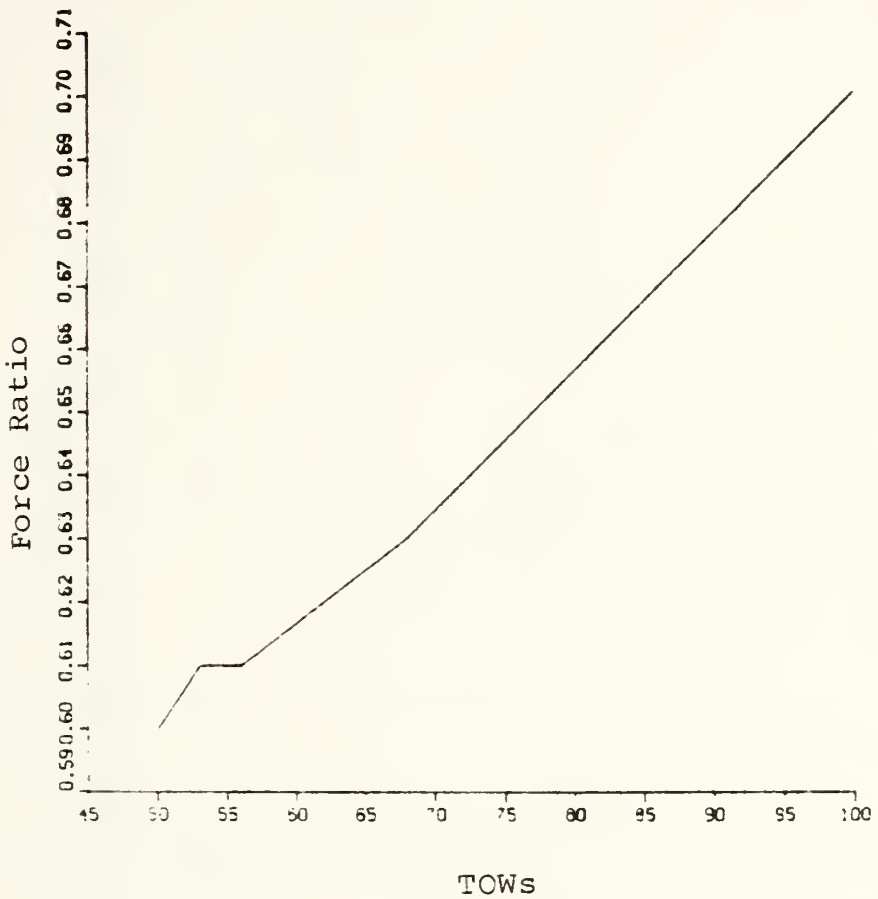

Figure 10. Force Ratio Versus TOWs

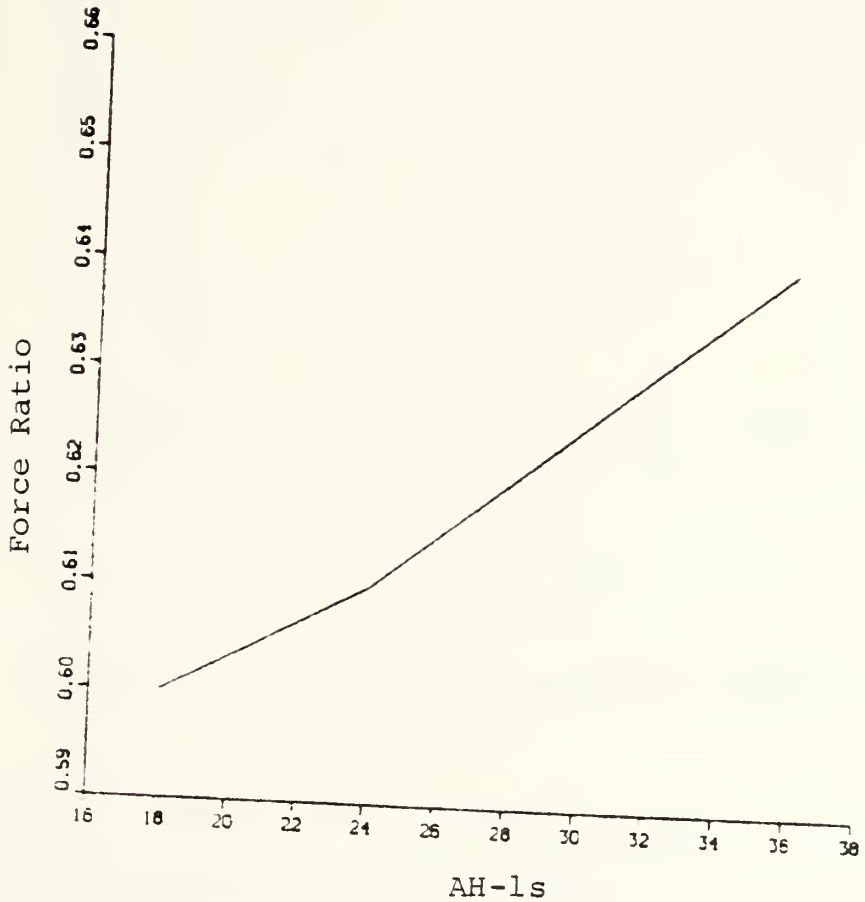

Figure 11. Force Ratio Versus AH-1s

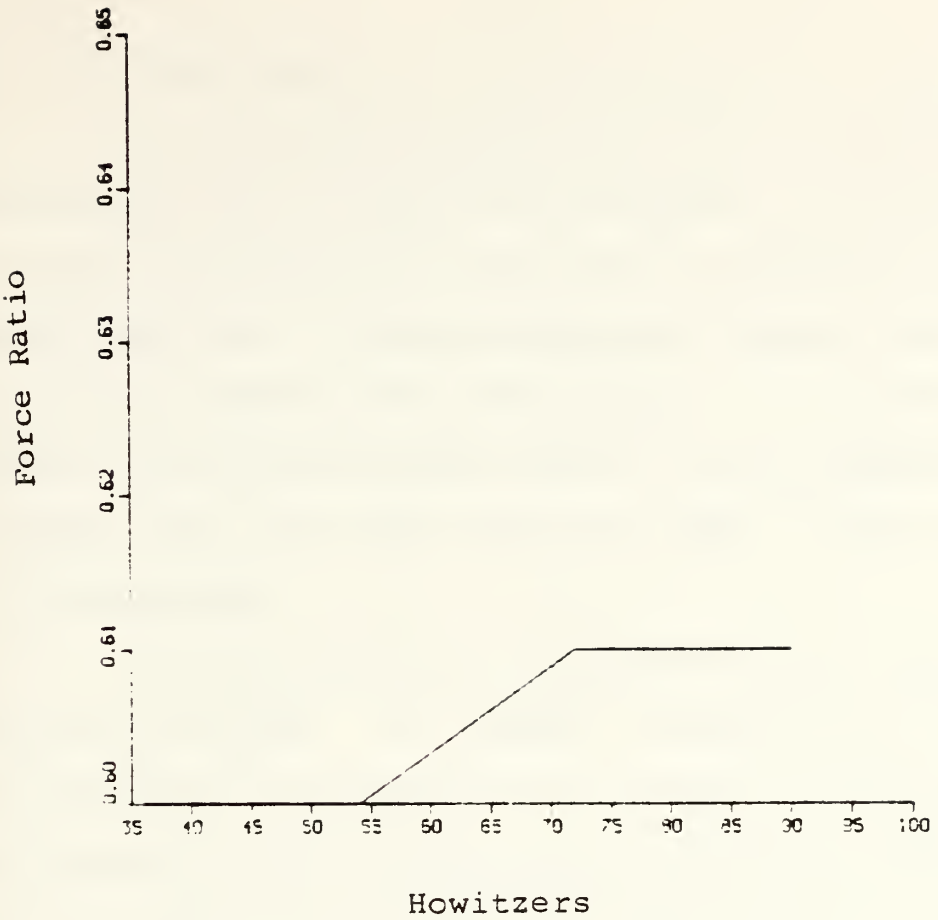

Figure 12. Force Ratio Versus Howitzers

Note that the rounding off of the force ratio makes the actual graph unknown for the howitzers.

b. Statistical Test for Linearity

Due to the results of the model runs, a statistical F test was run for each weapon to test if its relationship to the force ratio was linear. Appendix B contains the statistical F tests. The results of the test reveal that one rejects the insignificance of the linear coefficients hypothesis at the .05 and .01 significance levels. Thus the F test implies that linear is a good fit to the data.

c. Force Mix

In force mix analysis there are two methods: the predetermined and not predetermined methods. In the predetermined force mix questions, which will be referred to as the predetermined case, the user knows which weapons and what quantities are desired for investigation. In the not predetermined force mix questions, which will be referred to as the flagged case, the user allows the model to flag the weapon to be investigated.

The predetermined case questions are easily answered by the model. For example, suppose in our scenario it was critical to know whether to increase the tank force by one platoon or the artillery by one battery. Running the model with these changes shows the tanks improve the force ratio more than the artillery battery. Thus the recommendation would be made to increase tanks by one platoon.

The flagged case questions are not as easily answered. For example, suppose the commander wanted to improve his force ratio. The user would have to rerun the model with all possible force increases in order to answer the force mix question. The optimal solution may be omitted by user error and never achieved.

d. Trade Off Analysis

Through numerical analysis an investigation is made into the number of each blue weapon needed in order to increase the force ratio by the same amount. Our goal is to

determine if the quantity of weapons required to achieve a certain force ratio seem reasonable.

TABLE 17

Trade Off Analysis

ITEM	BASE QUANTITY	REQUIRED QUANTITY	DIFFERENCE	FORCE RATIO
Tanks	320	326	6	.61
TOW	50	53	3	.61
AH-1	18	24	6	.61
How	36	90	54	.61

As shown in the table, the relationship of the direct fire weapons (tank,TOW,AH-1) seem reasonable. The relationships says that a platoon of tanks is equal to a platoon of TOWs or a platoon of AH-1s.

It does not seem reasonable that a platoon of any of these direct fire weapons would be equal to a battalion plus of artillery. This is a significant flaw in the model's ability to examine force mix trade offs between direct and indirect fire weapons.

V. CONCLUSIONS

The Counterforce Potentials model has significant flaws which make it questionable for use in force mix analysis. The flaws discovered in the trade off analysis and the model's linear traits make the model suspect for use in the force mix analysis. The model seems not to display diminishing marginal returns which is a must for a model to be used for force mix analysis.

The model's strengths are in the following areas:

(1) Quantifying for each weapon type, more battlefield interrelationships within the P_{ij} and the $\hat{P}_{ji}$ matrices than the previous firepower score or AP-P models. These interrelationships include the calculation of K_{ij} with its attributes and the use of the indirect submodules to calculate E_i which are both used to calculate the P_{ij}'s.

(2) The straightforward flow of the submodules.

(3) The Counterforce Potentials force ratio is consistent with other linear models thus showing no radical numerical results.

The model's weaknesses include:

(1) There does not exist a value for the indirect fire systems which can be realistically compared to the value of the direct fire systems.

(2) The battlefield characteristics are not all quantified in an accurate or believable fashion.

(3) The model does not allow the artillery to fire mass or concentration of fires at a target or target group.

(4) The indirect fire submodule yields results which are not intuitively plausible as shown in the trade off analysis between the direct and indirect weapons.

(5) The model tends to be linear in the relationship between changes in force ratio and weapon quantities thus the model will not flag foolish combinations of weapon mixes.

(6) The model does not exhibit diminishing marginal returns.

(7) The model excludes the foot soldier and his basic weapons.

(8) The model does not allow the direct fire weapons to engage the indirect fire weapons.

VI. RECOMMENDATIONS

In order to improve the model the following recommendations are suggested:

(1) A value which can be realistically compared to the direct fire systems needs to be developed for the indirect fire systems.

(2) The direct fire weapons need to be able to engage susceptible indirect fire weapons.

(3) Foot soldiers need to be included in the model.

(4) Degradation of rounds for both counterbattery and countermaneuver need to be included in the model.

(5) Replace the user specified susceptibility for counterbattery fires with the probability of detection and the probability of engagement given a detection, and modify the section appropriately.

(6) The indirect fire submodules should reflect the artillery's ability to mass fires on known large enemy force locations.

(7) The order of the indirect fire submodules must be examined and modified to reflect the tactics and artillery missions being employed in the scenario.

(8) Increase the fractional allocational options for the direct fire systems to include the fractional effectiveness factor of the opposing system (E_j) with the quantity of that weapon (Q_j) to use $E_j \times Q_j$ in lieu of just Q_j.

(9) Improve the model so the indirect fire systems display a more logical trade off analysis with the direct fire weapons.

The following are suggested areas of future investigations:

(1) The consistency of the computer program to the theoretical documentation.

(2) A complete numerical analysis for all the input parameters of the model to examine their effects on the outputs.

(3) Validate the model with data collected in major exercises such as Reforger.

APPENDIX A

COUNTERFORCE RESULTS

A. GENERAL

This appendix contains a summary of the Counterforce Potentials computer runs which were used to perform the numerical analysis in Chapter IV.

B. SCENARIO DATA

The scenario data is the same base case as shown in Chapter IV. The inputs are varied in each test run and the resulting values and force ratios are shown in this appendix.

C. TEST RESULTS

1. Howitzer Changes

In these runs the howitzers were varied from 36 (base case) to 42, 54, 72, and 90. These quantities were chosen to represent changes of one battery, one company, one battalion, and one battalion plus reinforcements.

a. Case: 42 Howitzers

Blue force value: 356.7

Red Force Value: 593.

Force Ratio: .60

The weapon values were:

TABLE 18

Howitzer (42)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.65	1.01	.26	.77
Quan.	320	50	18	540	108	24
Total	319	25	12	546	28	18

b. Case: 54 Howitzers

Blue force value: 356.7

Red force value: 589.9

Force ratio: .60

TABLE 19

Howitzer (54)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.01	.25	.77
Quan.	320	50	18	540	108	24
Total	319	25	12	544	27	18

c. Case: 72 Howitzers

The blue force value: 356.7

The red force value: 585.3

The force ratio: .61

TABLE 20

Howitzers (72)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.00	.25	.76
Quan.	320	50	18	540	108	24
Total	319	26	12	540	27	18

d. Case: 90 Howitzers

Blue force value: 356.7

Red force value: 580.7

Force ratio: .61

TABLE 21

Howitzer (90)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	.99	.24	.76
Quan.	320	50	18	540	108	24
Total	319	25	12	537	26	18

2. Tank Changes

In this section the tanks were varied by 6, 18, and 54. These correspond to increases by a platoon, a company, and a battalion of tanks. Tanks were also run at 640, 1000 and 10000.

a. Case: 326 Tanks

Blue force value: 362.7

Red force value: 592.5

Force ratio: .61

TABLE 22

Tanks (326)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.01	.26	.76
Quan.	326	50	18	540	108	24
Total	325	26	12	546	28	18

b. Case: 338 Tanks

Blue force value: 374.8

Red force value: 587.9

Force ratio: .64

TABLE 23

Tanks (338)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.0	.51	.66	1.0	.26	.75
Quan.	338	50	18	540	108	24
Total	337	26	12	542	287	18

c. Case: 374 Tanks

Blue force value: 410.8

Red force value: 573.3

Force ratio: .72

TABLE 24

Tanks (374)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	.98	.25	.72
Quan.	374	50	24	540	108	24
Total	373	26	12	529	27	17

d. Case: 640 Tanks

The blue force value: 670.2

The red force value: 484.8

The force ratio: 1.38

TABLE 25

Tanks (640)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.43	.56	.83	.20	.58
Quan.	640	50	24	540	108	24
Total	639	22	10	449	22	14

e. Case: 1000 Tanks

The blue force value: 1027.2

The red force value: 473.3

The force ratio: 2.17

TABLE 26

Tanks (1000)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.40	.52	.81	.20	.56
Quan.	1000	50	24	540	108	24
Total	998	20	9	438	21	13

f. Case: 10000 Tanks

The blue force value: 10006.4

The red force value: 547.9

The force ratio: 18.26

TABLE 27

Tanks (10000)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.39	.50	.94	.23	.63
Quan.	10000	50	24	540	108	24
Total	9978	19	9	508	25	15

3. TOW Changes

The TOW was varied by 3, 6 and 18. These represent changes in TOWs of a section, company, and a squadron of an armored cavalry squadron. The TOWs were also doubled to 50.

a. Case: 53 TOWs

Blue force value: 358.2

Red force value: 591.6

Force ratio: .61

TABLE 28

TOW (53)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.00	.26	.76
Quan.	320	53	18	540	108	24
Total	319	29	12	542	28	18

b. Case: 56 TOWs

Blue force value: 359.8

Red force value: 588.7

Force ratio: .61

TABLE 29

TOW (56)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.00	.26	.76
Quan.	320	56	18	540	108	24
Total	319	29	12	542	28	18

c. Case: 68 TOWs

Blue force value: 366.1

Red force value: 577.5

Force ratio: .63

TABLE 30

TOW (68)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	.99	.25	.75
Quan.	320	68	18	540	108	24
Total	319	35	12	533	27	18

d. Case: 100 TOWs

Blue force value: 382.7

Red force value: 550.5

Force ratio: .70

TABLE 31

TOW (100)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.67	.94	.23	.72
Quan.	320	100	18	540	108	24
Total	319	51	12	508	25	17

4. AH-1 Changes

The AH-1 helicopter is varied by 6 and 18. These represent changes of a platoon and a company size.

a. Case: 24 AH-1s

Blue force value: 360.8

Red force value: 589.4

Force ratio: .61

TABLE 32

AH-1 (24)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.66	1.00	.27	.77
Quan.	320	50	24	540	108	24
Total	319	26	16	542	29	19

b. Case: 36 AH-1s

Blue force value: 369.1

Red force value: 579.2

Force ratio: .64

TABLE 33

AH-1 (36)

	M60	ITH	AH-1	T72	BMP	Hind
Value	1.00	.51	.67	.98	.27	.77
Quan.	320	50	36	540	108	24
Total	319	26	24	531	29	19

APPENDIX B

STATISTICAL ANALYSIS

A. GENERAL

For each weapon a statistical F test was run on the hypothesis that the relationship between the weapon increases and the force ratio was linear [11].

B. STATISTICAL TESTS

1. Hypothesis Test for Linearity

a. Hypothesis (H_0)

All the parameters are insignificant (no relationship between data).

b. Alternate Hypothesis (H_1)

Some parameters are not insignificant (model is linear).

c. Critical Region

Reject the null hypothesis (H_0) if F is greater than or equal to F_{α,ν_1,ν_2}

2. F Statistics (From Tables)

TABLE 34

F Statistics

ITEM	DF	$F_{.01}$	$F_{.05}$
Tanks	(5,5)	8.47	4.28
TOW	(4,4)	16.0	6.39
AH-1	(3,3)	29.5	9.28
How	(4,4)	16.0	6.39

a. TANKS

Letting Y be a vector of the force ratio for tank changes and X be a vector of the quantity of tanks resulting in the force ratio change, a regression was run on Y versus X.

TABLE 35

Regression Force Ratio vs Tanks

Source	DF	Sum Squares	Mean Square	F-STAT.
regression	1	2.566E2	2.566E2	1.287E4
residual	5	9.972E-2	1.994E-2	
total	6	2.5679E2		

R square: .999611

Std error: .141224

coefficient: .1087 t-stat: 1.7785

coefficient: .0018 t-stat: 113.45

Reject H_0 since 1287 is greater than F_{α,ν_1,ν_2}

b. TOWs

Using the same format, the regression yielded:

TABLE 36

Regression Force Ratio vs TOW

Source	DF	Sum Square	Mean Square	F-STAT.
regression	1	6.55E-3	6.55E-3	381.3
residual	3	5.15E-5	1.717E-5	
total	4	6.6E-3		

R square: .9922

Std error: .0041

coefficients: .501 and .002, t-stats 73.02 and 19.53

Reject H_0 since 381.3 is greater than F_{α,ν_1,ν_2}

c. AH-1

Using the same format the regression yields:

TABLE 37

Regression Force Ratio vs AH-1

Source	DF	Sum Squares	Mean Square	F-STAT.
regression	1	8.59E-4	8.58E-4	120.3
residual	1	7.14E-6	7.14E-6	
total	2	8.66E-4		

R square: .992

Std error: .0026

coefficients: .5579 and .0023, t-stats 99.99 and 10.97

Reject H_0 since 120.32 is greater than F_{α,ν_1,ν_2}

d. Artillery

Using the same format the regression yields:

TABLE 38

Regression Force Ratio vs Artillery

Source	DF	Sum Square	Mean Square	F-STAT.
regression	1	6.78E-5	6.78E-5	18.89
residual	2	7.18E-6	3.589E-6	
total	3	7.5E-5		

R square: .9042

Std error: .00189

coefficients: .592 and .0002 with t-stats 220.5 and 4.34

Reject H_0 since 18.89 is greater than F_{α,ν_1,ν_2}

C. SUMMARY

In each case the testing yielded a large value for the F statistic which indicated rejection of the null hypothesis (H_0). The fact that some parameters may be significant implies that one must accept linearity at the .01 and .05 significance levels.

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